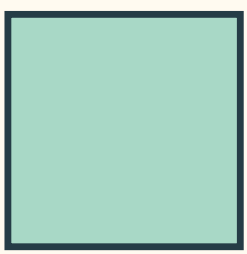
MATHS FIELD GUIDE · GRADE 10

# Writing and critiquing a geometric proof

OPENING PROBLEM

## If a quadrilateral is a square, are its diagonals equal?



square

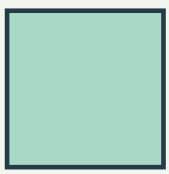
02 Test an implication in both directions
03 Rules and worked cases
04 From the givens to the answer
05 A parallel cut creates two similar triangles
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

# Test an implication in both directions

A proof uses the stated conditions. A counterexample can defeat a claim that says “always”.

1



square

### If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

2

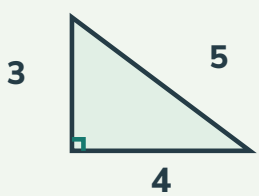


rectangle

### If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

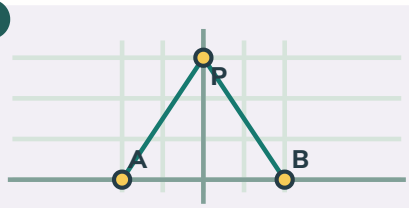
3




### Use one diagram to distinguish a theorem from its converse.

A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

4




A (-2, 0)
P (0, 3)
B (2, 0)

### Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

**A single counterexample**
**defeats an “always” claim.**

**A single matching drawing**
**illustrates the claim but cannot prove it in general.**

CHAPTER PROBLEM

### Prove that a line through the midpoints of AB and AC in ΔABC is parallel to BC.

Both upper-to-lower ratios are 1. Apply the converse of BPT to obtain parallelism.

MATHS REFERENCE · GRADE 10

# Writing and critiquing a geometric proof

## Rules, conditions and worked cases

| Recognise the case | Mathematical reasoning |
|---|---|
| **Name the condition** | To use alternate interior angles as equal, first establish that the two lines are parallel. |
| **Keep correspondence** | If ΔABC ~ ΔDEF, pair AB with DE and BC with EF. Mixing DE and DF can reverse the conclusion. |
| **Expose circular reasoning** | To prove DE ∥ BC from equal ratios, use the converse of BPT; starting with equal corresponding angles assumes what must be shown. |

## Read the representations

### A proof connects given facts to the conclusion

Write givens separately from what is to be shown. A diagram records relationships; an unmarked visual impression is not a premise.

Pair each statement with its reason: common angle, parallel-angle relationship, similarity criterion, or corresponding side proportion.

A complete chain

| Statement | Reason |
|---|---|
| **DE∥BC** | Given |
| **∠ADE=∠ABC and ∠AED=∠ACB** | Corresponding angles |
| **ΔADE~ΔABC** | AA |
| **AD/AB=AE/AC** | Corresponding sides |

### Repair the first unsupported step

If the proof assumes the target, later correct algebra cannot rescue it.

If using SAS similarity, the equal angle must lie between the two proportional sides.

| Triangle | Side 1 | Side 2 | Side 3 |
|---|---|---|---|
| ABC | 3 | 4 | 5 |
| PQR | 3 | 4 | 5 |

### Can all three matching side lengths establish congruence?

Yes, by SSS. Match the vertex opposite each corresponding side.

WORKED EXAMPLES

# From the givens to the answer

### Critique “the triangles look the same, so their sides are proportional”

1. Appearance does not prove similarity.
2. First identify equal angles from a shared angle and a stated parallel line.
3. Conclude AA, then use the correctly ordered side ratios.

### Find the gap in a proof

A pupil says that two triangles with sides 3, 4 and 6, 8 must be similar. What extra information would finish the proof?

1. The two known ratios agree, but the angle between those sides must also agree for SAS similarity. Equal included right angles would give matching 3–4–5 and 6–8–10 triangles.

## Check another case

### Example 1

**point arrangement**

| Point | D | E |
|---|---|---|
| Location | Between A and B | Between A and C |

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

If AD/DB=2/3 and AE=6, find EC when DE∥BC.

9

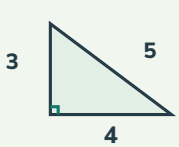
Apply BPT in consistent order.

### Example 2

A proof establishes $\Delta ABC \sim \Delta DEF$ with $AB/DE=3/5$ and $EF=45$. Find BC.

27

BC corresponds to EF.




### Two triangles each have legs 3 and 4 meeting at 90°. Is that enough?

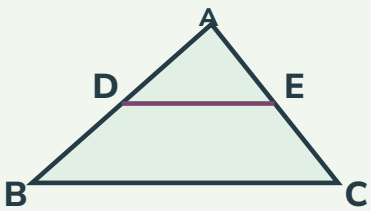
Yes. Two corresponding sides and their included right angle give SAS.

ILLUSTRATED PROBLEM SHEET

# A parallel cut creates two similar triangles

D lies on AB, E lies on AC and DE is parallel to BC unless the question asks you to test it.

1




DE ∥ BC

## Use AB = 8, AC = 6. AD = 4. Find AE.

D is halfway to B. The smaller triangle is at half scale, so AE = 3.

2

AD = 3, DB = 6
↓
AE = 4
↓
AD/DB = AE/EC

## Recover the missing segment EC.

The lower piece is twice the upper piece on AB. EC must be twice 4, which is 8.

3

| Side | Upper | Lower |
|---|---|---|
| AB | 3 | 5 |
| AC | 6 | 10 |

## If parallelism was not given, can you deduce it?

Yes. 3/5 = 6/10. The converse applies because D and E are internal points.

4

AD/AB = 2/5
↓
Area ADE / Area ABC = 4/25

## What fraction of the large area remains below DE?

1 − 4/25 = 21/25. The area factor is the square of the length factor.

**AD:DB = 1:3**
**AD:AB = 1:4.**

**AD:AB = 3:4**
**AD:DB = 3:1.**

CHAPTER PROBLEM

**A proof establishes ΔABC∼ΔDEF with AB/DE=3/5 and EF=25. Find BC.**

15. BC corresponds to EF.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

**Data A**

**point arrangement**

| Point | D | E |
|---|---|---|
| Location | Between A and B | Between A and C |

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

| Try the problem | Solution and reasoning |
|---|---|
| **01**<br>Can measuring one drawing prove a relationship for every triangle? | No. State the conditions and derive the relationship for an arbitrary triangle. |
| **02**<br>May you set AD/DB equal to AE/AC for a parallel cut? | Those fractions compare different parts. Match part to remainder on both sides, or part to whole on both. |
| **03**<br>Which reason justifies equal vertical angles?<br>A) The lines are parallel.<br>B) The angles are formed by two intersecting straight lines.<br>C) The triangles look equal.<br>D) The target says so. | The angles are formed by two intersecting straight lines.<br>Vertical angles arise at a line intersection. |
| **04 · Data A**<br>If AD/DB=2/3 and AE=16, find EC when DE∥BC. | 24<br>Apply BPT in consistent order. |
| **05**<br>A learner applies SAS similarity using a non-included equal angle. Explain the gap. | SAS requires the equal angle between the proportional sides.<br>A non-included angle does not generally determine similarity; seek another valid criterion. |
| **06**<br>Which is an allowed proof starting point?<br>A) A statement to be proved.<br>B) A conclusion from an unscaled sketch.<br>C) An explicitly supplied parallel relationship.<br>D) A hoped-for ratio. | An explicitly supplied parallel relationship.<br>A given relationship is valid evidence. |

**Equal angles: 40°, 60°, 80°**
↓
**Same shape**
↓
**Size still free**

### Do three equal angle pairs prove congruence?

They give similarity. One triangle may be an enlargement of the other.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **07**<br>Explain why the midpoint theorem follows from similarity. | Use the midpoint ratios and BPT converse to get parallelism.<br>Then AA gives $\Delta ADE \sim \Delta ABC$ with side factor 1/2; hence $DE = BC/2$. |
| **08**<br>How should you check a proportional statement before cross-multiplying? | Name the matched vertices and ensure denominators are nonzero lengths.<br>Keep all side ratios in the same direction and distinguish full sides from subsegments. |
| **09**<br>A proof establishes $\Delta ABC \sim \Delta DEF$ with $AB/DE = 3/5$ and $EF = 30$. Find BC. | 18<br>BC corresponds to EF. |
| **10**<br>A proof establishes $\Delta ABC \sim \Delta DEF$ with $AB/DE = 3/5$ and $EF = 35$. Find BC. | 21<br>BC corresponds to EF. |
| **11**<br>A proof establishes $\Delta ABC \sim \Delta DEF$ with $AB/DE = 3/5$ and $EF = 40$. Find BC. | 24<br>BC corresponds to EF. |

| Third side | 3 | 7 | 11 |
|---|---|---|---|
| Other sides | 4 and 7 | 4 and 7 | 4 and 7 |
| Triangle? | no | yes | no |

### Why do endpoints fail?

The third side must be strictly between 3 and 11. Equality collapses the triangle into a straight line.

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