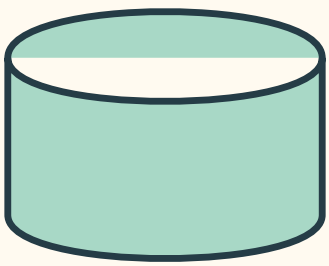
MATHS FIELD GUIDE · GRADE 10

Volumes of combined solids

OPENING PROBLEM

Only the curved outside receives paint at ₹2 per square unit. What does it cost?



r = 3, h = 4

02 Sketch the join before choosing add or subtract
03 Rules and worked cases
04 From the givens to the answer
05 Match the solid to the formula
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Sketch the join before choosing add or subtract

Volume follows the material present. Surface area follows the boundary exposed.

1

Cylinder r = 3, h = 4
↓
Hemisphere r = 3
↓
36π + 18π

A dome sits on the cylinder. Find the combined volume.

54π cubic units. The solids share a face and have no overlapping volume.

2

Same cylinder and dome
↓
Wall 24π + dome 18π
↓
Base 9π

Find the external area including the bottom.

51π square units. The matching top and bottom disks at the join are hidden.

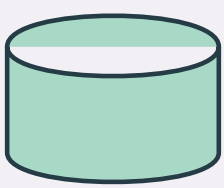
3

Cylinder r = 3, h = 6
↓
Matching cone removed
↓
54π − 18π

How much material remains after drilling the cone?

36π cubic units. A void subtracts volume.

4



r = 3, h = 4

Only the curved outside receives paint at ₹2 per square unit. What does it cost?

The wall is 24π square units. Cost is ₹48π.

Hollow out a cavity
material volume decreases.

An accessible cavity
exposed surface area may increase.

CHAPTER PROBLEM

A cylinder radius 3, height 5, has a hemisphere radius 3 attached. Write total volume as cπ. Find c.

63. Add 9hπ and 18π.

MATHS REFERENCE · GRADE 10

Volumes of combined solids

Rules, conditions and worked cases

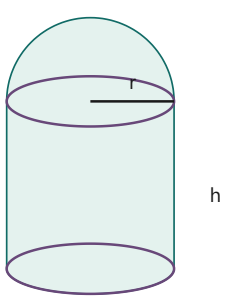
Recognise the case	Mathematical reasoning
Cylinder plus hemisphere	With common r = 2 and cylinder height 6, volume = 24π + 16π/3 = 88π/3.
Cavity	A cone cut from a cylinder of the same base and height removes one third. Two thirds of the cylinder volume remain.
Melt and recast	If no material is lost, equate volumes. One sphere of radius 6 provides eight spheres of radius 3.

Read the representations

Add disjoint parts; subtract a removed part

A cylinder with a hemisphere attached is modelled as two solids whose interiors do not overlap. Volumes add.

A cone-shaped cavity is subtracted from the containing solid. Use internal dimensions for capacity. The shared boundary has zero volume.

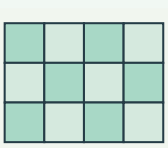


The circular join is internal, so omit it from outside area.

Match interfaces and units before calculating

For a cone attached to a cylinder on the full circular face, their radii must match. The total height splits into component heights.

Keep exact π until a requested estimate; 1 cm³=1 mL and 1000 cm³=1 L. Combinations here use two permitted solids.



One layer of unit cubes forms this grid. How many cubes?

Three rows of four make 12 cubes.

WORKED EXAMPLES

From the givens to the answer

Cylinder plus hemisphere, r=3 cm and cylinder h=8 cm

1. Cylinder volume=$\pi(3^2)(8)=72\pi$ cm³.
2. Hemisphere volume=$(2/3)\pi(3^3)=18\pi$ cm³.
3. Total=90π cm³. Overall height is 8+3=11 cm, not the cylinder height.

A cone removed from a cylinder

1. Both have radius 3 and height 6. Cylinder volume is 54π; cone volume is 18π.
2. Remaining material volume is 36π cubic units, assuming the cone lies wholly inside the cylinder.

Remove a cone from a cylinder

A cone is hollowed out of a cylinder with the same radius 3 cm and height 8 cm. How much solid remains?

1. The cylinder occupies 72π cm³. The cone occupies one third of that, 24π cm³. Subtract the missing material to obtain 48π cm³.

Check another case

Example 1

A cone radius 3, height 9 is removed from a cylinder of the same radius and height. Write remaining volume as $c\pi$. Find c.

54

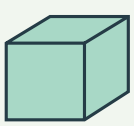
The cylinder contains $9 \times 9\pi = 81\pi$ cubic units. The cone occupies one third, or 27π, leaving 54π.

Example 2

A cylinder is topped by a hemisphere of radius 4. Total height is 14. Find cylinder height.

10

Hemisphere height equals its radius.



side 3 cm

How many 1 cm cubes fill this cube?

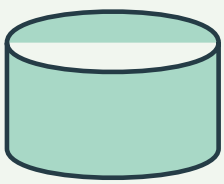
Nine fit in a layer. Three layers give 27 cm³.

ILLUSTRATED PROBLEM SHEET

Match the solid to the formula

Volume uses perpendicular height. A slant length is useful for a curved surface or a triangular face.

1

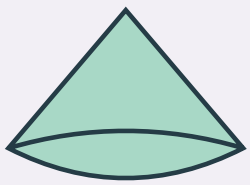


r = 3, h = 8

Find the volume of the cylinder.

Base area 9π times height 8 gives 72π cubic units.

2

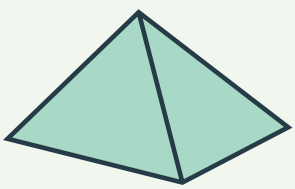


r = 3, h = 8

How much of the matching cylinder does this cone hold?

One third, or 24π cubic units. The base and perpendicular height must match.

3



B = 18, h = 5

Find the volume without using a slant length.

Bh/3 = 30 cubic units. B is the area of the base.

4



r = 3

Find the volume of a hemisphere of this radius.

Half of (4/3)π × 27 is 18π cubic units.

Prism volume V and same base and height pyramid
V/3.

A cylinder of radius r, height 2r
volume $2\pi r^3$.

CHAPTER PROBLEM

A cone radius 3, height 6 is removed from a cylinder of the same radius and height. Write remaining volume as cπ. Find c.

36. The cylinder contains 9 × 6π = 54π cubic units. The cone occupies one third, or 18π, leaving 36π.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 If a hemisphere is detached from its matching cylinder, how much exposed area appears?	Two previously hidden circular faces appear. Together they add $2\pi r^2$.
02 Two cones have equal heights but one has twice the radius. How do volumes compare?	Base area grows fourfold, so volume grows fourfold as well.
03 A cylinder is topped by a hemisphere of radius 4. Total height is 12. Find cylinder height.	8 Hemisphere height equals its radius.
04 A cylinder and cone share radius 3; cylinder height 9, cone height 6. Write combined volume as $c\pi$. Find c.	99 Cylinder $9h\pi$ plus cone 18π.
05 Which dimensions determine a vessel's capacity? A) External dimensions always B) Internal dimensions C) Slant height only D) Painted area	Internal dimensions Capacity measures the available inside volume.
06 Convert 4000 cm^3 into litres.	4 1000 cm^3=1 L.

Layers	Volume (cm^3)
1	12
2	24
5	60

Which box has volume 60 cm^3 if its base is 12 cm^2?

The height is 5 cm. Divide the volume by the base area.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why a shared circular face is not subtracted from the sum of component volumes.	A face has area but zero three-dimensional volume. The component interiors are disjoint in the joined-solid model.
08 A cylinder of radius r and height h has a cone-shaped cavity of the same r and h. Derive the remaining volume.	Subtract $\pi r^2 h/3$ from $\pi r^2 h$. The result is $2\pi r^2 h/3$; the cone must fit inside the cylinder.
09 A cylinder radius 3, height 6, has a hemisphere radius 3 attached. Write total volume as $c\pi$. Find c.	72 Add $9h\pi$ and 18π.
10 A cone radius 3, height 7 is removed from a cylinder of the same radius and height. Write remaining volume as $c\pi$. Find c.	42 Subtract $3h\pi$ from $9h\pi$.
11 A cylinder radius 3, height 8, has a hemisphere radius 3 attached. Write total volume as $c\pi$. Find c.	90 The cylinder contributes $3^2 \times 8\pi = 72\pi$. The hemisphere contributes $(2/3) \times 3^3\pi = 18\pi$. Together they occupy 90π cubic units.

240 cm³
↓
240 mL
↓
0.24 L

How does water help compare capacity and volume?

One cubic centimetre of internal space holds one millilitre.

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