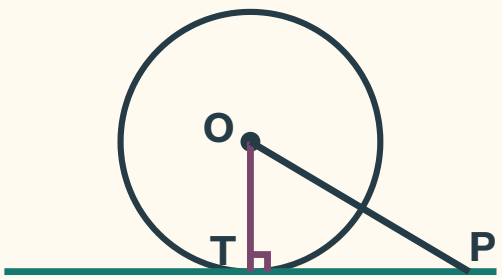
MATHS FIELD GUIDE · GRADE 10

Tangent and radius theorem

OPENING PROBLEM

OT = 5 cm and OP = 13 cm. How long is PT?



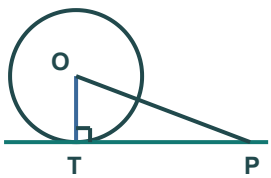
OT ⊥ PT

02 Rules and worked cases
03 Find the right angle at contact
04 Recognise the conditions
05 From the givens to the answer
06 Which relationship proves it?
07 Draw the perpendicular to a chord
08 Check your reasoning
09 Check your reasoning

CIRCLE GEOMETRY · ILLUSTRATED PROBLEMS

Tangent and radius theorem

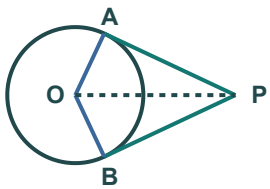
One contact point



OT = 8, OP = 17. Find PT.

The radius meets the tangent at 90°. $PT^2 = 17^2 - 8^2 = 225$, so PT = 15.

Two tangents



PA = 2x + 3 and PB = 17. Find x.

Right triangles OAP and OBP share OP and have equal radii. RHS gives PA = PB, so x = 7.

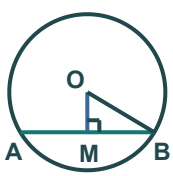
Test a line



A circle has radius 9. Which lines touch it?

Measure perpendicular distance from O. Distance 7 gives two crossings, 9 gives one contact, and 11 misses.

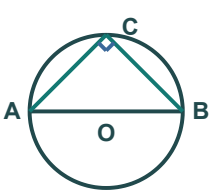
Bisect a chord



Radius 13, OM = 5. Find AB.

The perpendicular from O bisects AB. $MB = \sqrt{(13^2 - 5^2)} = 12$, so AB = 24.

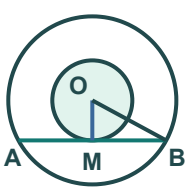
Use a diameter



AB is a diameter. Why is angle ACB 90°?

The intercepted semicircle measures 180°. Its inscribed angle is half that measure.

Use two radii



Radii 10 and 6. Find the outer chord AB.

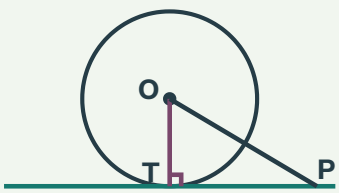
OM is perpendicular to AB at contact with the inner circle. Half-chord $= \sqrt{(10^2 - 6^2)} = 8$. AB = 16.

ILLUSTRATED PROBLEM SHEET

Find the right angle at contact

At the point of tangency the radius is perpendicular to the tangent line.

1

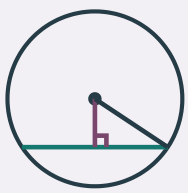


OT ⊥ PT

OT = 5 cm and OP = 13 cm. How long is PT?

OP is the hypotenuse. PT = √(169 − 25) = 12 cm.

2



r = 5, d = 3

How can distance to a line distinguish a secant from a tangent?

Three is less than the radius five. The line crosses twice. Half the chord is four.

3

PA = PB

↓

3x − 2 = x + 8

↓

x = 5

Tangents from one external point have these expressions. Find their lengths.

Both equal 13. They run from the same point to different contacts on the same circle.

4

∠OAP = ∠OBP = 90°

↓

∠APB = 54°

↓

∠AOB = 126°

Why do the last two angles sum to 180°?

The right angles already use 180° of quadrilateral OAPB's 360°.

Centre distance to line below r
two intersections.

Centre distance to line above r
no intersections.

CHAPTER PROBLEM

PT is tangent at T, OT=9, OP=15. Find PT.

12. The radius is perpendicular at contact. Subtract the squared radius from the squared centre distance: $15^2 - 9^2 = 144$, then take the square root.

RULES AND EXAMPLES

Recognise the conditions

Rules, conditions and worked cases

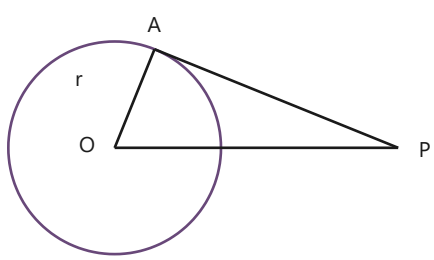
Recognise the case	Mathematical reasoning
Contact angle	At a tangent point T, OT ⊥ PT. Triangle OTP is right at T; OP is its hypotenuse.
Recover a tangent length	Radius 7, OP = 25 give PT = √(625 − 49) = 24.
Test a candidate line	Minimum distance d from the centre decides: d < r gives a secant, d = r a tangent, d > r no intersection.

Read the representations

A tangent meets a circle at exactly one point

At contact T, the radius OT is perpendicular to the tangent. A secant meets the circle at two points.

The tangent theorem supplies the right angle needed for Pythagoras in an external-point triangle.



Radius ⊥ tangent at contact

Why the radius must be perpendicular

Let OM be the perpendicular from the centre to the tangent line. If M≠T, OM<OT=r.

A line at distance less than r cuts a circle twice: the two intersection positions along the line have distances $\pm\sqrt{(r^2-OM^2)}$ from M.

That contradicts a tangent having only one intersection. Therefore M=T and OT is perpendicular to the tangent.

1. **Suppose the foot is not T**OM<OT=r
2. **Distance below radius**The line has two circle intersections
3. **Contradiction**A tangent has exactly one
4. **Conclude**OT ⊥ tangent at T



r = 5

What is the diameter?

Two radii laid end to end through the centre give 10.

WORKED EXAMPLES

From the givens to the answer

Find a tangent length

1. OT=5 cm, OP=13 cm and PT is tangent at T.
2. ∠OTP=90°, so $PT^2=13^2-5^2=144$.
3. PT=12 cm; the positive root is the length.

Is this line really a tangent?

O is a circle's centre and T is on it. OT = 8, PT = 15 and OP = 17. Is PT tangent at T?

1. Since $8^2 + 15^2 = 289 = 17^2$, the converse of Pythagoras gives a right angle at T. A line perpendicular to the radius at its endpoint on the circle is a tangent.

Check another case

Example 1

OT=40,PT=96. Find OP.

104

Use the right triangle: $OP^2=OT^2+PT^2$.

Example 2

A line is tangent to a circle of radius 6. Find the perpendicular distance from centre to line.

6

The perpendicular from centre ends at contact.

Can a line one unit closer still touch once?

In a circle of radius 5 cm a line lies 4 cm from the centre. The perpendicular from the centre halves the chord. Half its length is $\sqrt{(25 - 16)} = 3$ cm.

The line cuts a 6 cm chord. Sliding it out to distance 5 cm shrinks the chord to a single contact point.



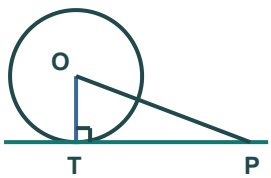
Which chord is longest?

A diameter. Its length is twice the radius, here 12.

CIRCLE GEOMETRY · PROOF AND RECONSTRUCTION

Which relationship proves it?

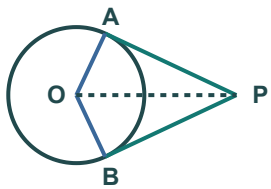
Prove perpendicularity



Why must the radius to T meet the tangent at a right angle?

If the perpendicular foot H were elsewhere on the tangent, OH would be shorter than OT. H would be inside the circle, making the line cross it twice.

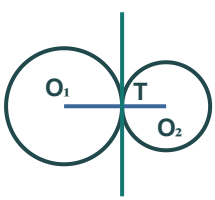
Bisect an external angle



Why does OP split angle APB equally?

OAP and OBP are right triangles with equal radii and common hypotenuse OP. RHS congruence gives equal angles at P.

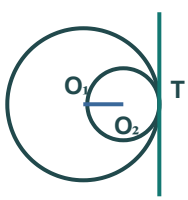
Touching externally



Radii are 7 cm and 4 cm. Find the distance between the centres.

Both radii are perpendicular to the common tangent at T, so they lie on one straight line. T is between the centres. Add: 7 + 4 = 11 cm.

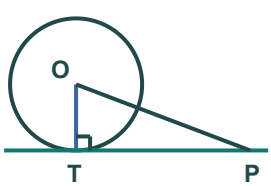
Touching internally



The same circles now touch internally. What is the centre distance?

Both radii run from T to the same side of the tangent. The distance between centres is the excess of the longer radius: 7 − 4 = 3 cm.

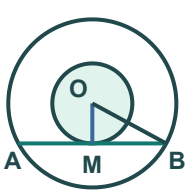
Recover the centre distance



A tangent segment is 20 cm. The radius is 15 cm. Is the centre 35 cm from P?

OP is the hypotenuse, so $OP^2 = 20^2 + 15^2 = 625$ and OP = 25 cm. The sum 35 cm follows the broken path P–T–O.

Reconstruct a radius



The outer radius is 10 cm. A 12 cm outer chord touches the inner circle. What is the inner radius?

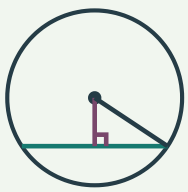
Contact M bisects the chord. The right triangle has hypotenuse 10 and one leg 6. The other leg OM = $\sqrt{(100 - 36)} = 8$ cm.

ILLUSTRATED PROBLEM SHEET

Draw the perpendicular to a chord

A perpendicular from the centre to a chord bisects it. The resulting right triangle uses half the chord.

1

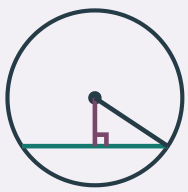


r = 10, d = 6

Find the length of the chord.

Half-length = √(100 − 36) = 8. Double it to get 16.

2

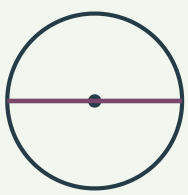


r = 13, AB = 10

How far from the centre is chord AB?

Half the chord is 5. Distance = √(169 − 25) = 12.

3



d = 0

Which chord has its midpoint at the centre?

A diameter. Zero distance leaves half-chord equal to the radius.

4

Radius	Distance	Chord
5	0	10
5	3	8
5	4	6

How does moving a chord away from the centre affect it?

Within the same circle the chord shortens. The right triangle has a fixed hypotenuse.

Radius 5, distance 4
chord length 6.

Radius 5, distance 3
chord length 8.

CHAPTER PROBLEM

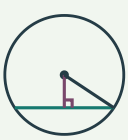
Find the angle in degrees between a radius and its tangent at contact.

90. The radius is perpendicular to the tangent.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can a point inside a circle have a tangent segment to the circle?	No. Its centre distance is less than the radius. Every line through it intersects the circle twice.
02 Two chords lie 4 cm from the centres of different circles. Must their lengths match?	No. The radii must also match for the half-chord calculations to agree.
03 A circle has centre O. PT touches it at T. If OT = 30 cm and PT = 72 cm, find OP in centimetres.  	78 The radius OT is perpendicular to the tangent PT. Hence $OP^2 = 30^2 + 72^2 = 6{,}084$ and OP = 78.
04 Which line is a tangent? A) A line meeting a circle at two points. B) A line with exactly one point of intersection with the circle. C) Any line through the centre. D) Any line outside the disk.	A line with exactly one point of intersection with the circle. A tangent has one contact point.
05 A line is tangent to a circle of radius 11. Find the perpendicular distance from centre to line.	11 The perpendicular from centre ends at contact.
06 OP=26,OT=10. Find PT^2.	576 Subtract squared radius from squared centre-distance.



A radius is 13 and its centre is 5 from a chord. Find the chord length.

The perpendicular halves the chord. Half-length = √(169 − 25) = 12. Full length = 24.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Prove the radius-tangent theorem using the perpendicular from the centre to the tangent.	Assume its foot differs from the contact point, giving centre-line distance less than r. Such a line intersects the circle twice, contradicting tangency. The foot is the contact point, proving perpendicularity.
08 Why can you not apply this right angle to every chord endpoint?	A chord lies on a secant, which has two circle intersections. The perpendicularity requires a tangent at the radius endpoint.
09 PT is tangent at T, OT=12, OP=20. Find PT. 	16 The radius is perpendicular at contact. Subtract the squared radius from the squared centre distance: $20^2 - 12^2 = 256$, then take the square root.
10 PT is tangent at T, OT=15, OP=25. Find PT. 	20 The radius is perpendicular at contact. Subtract the squared radius from the squared centre distance: $25^2 - 15^2 = 400$, then take the square root.
11 PT is tangent at T, OT=18, OP=30. Find PT. 	24 The radius is perpendicular at contact. Subtract the squared radius from the squared centre distance: $30^2 - 18^2 = 576$, then take the square root.



Which part is one quarter of the circumference?

The curved arc between the two radii. The filled sector is one quarter of the disk area.

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