MATHS FIELD GUIDE · GRADE 10

Statistics and probability case-study workshop

OPENING PROBLEM

Estimate the mean using all ten observations.

Class	0–10	10–20	20–30
Frequency	2	6	2
Midpoint	5	15	25

02 Read the frequencies before estimating
03 Rules and worked cases
04 From the givens to the answer
05 Decide which population a claim describes
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Read the frequencies before estimating

For grouped values, a midpoint, an interpolation or a frequency peak answers a different question.

1

Class	0–10	10–20	20–30
Frequency	2	6	2
Midpoint	5	15	25

Estimate the mean using all ten observations.

Weighted total = 2 × 5 + 6 × 15 + 2 × 25 = 150. Divide by ten for 15.

2

Class	0–10	10–20	20–30
Frequency	4	10	6
Cumulative	4	14	20

Where does the median lie?

Half the total is 10. Interpolate six tenths of the way across the middle class: 10 + 6 = 16.

3

Class	0–10	10–20	20–30
Frequency	3	9	6

Estimate the mode of these equal-width classes.

The rise to the peak is 6 and the fall is 3. Estimate 10 + 10 × 6/9 = 16⅔.

4

Midpoint	10	20	30
Frequency	2	1	3
Step code	−1	0	1

Choose centre 20 and step 10. Estimate the mean.

Weighted code sum = 1. The mean is 20 + 10/6 = 65/3.

Halve every frequency proportionally grouped mean unchanged.

Add five to every class boundary grouped mean rises by five.

CHAPTER PROBLEM

Why can the grouped mean differ from the exact mean of the original data?

The calculation puts every observation at its class midpoint. Their actual positions are unknown.

MATHS REFERENCE · GRADE 10

Statistics and probability case-study workshop

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Frequency to chance	For a uniformly chosen record from groups with counts 3,7,10, the middle group has probability 7/20.
Frequency to a mean	For corresponding class midpoints 5,15,25, estimated mean = (15 + 105 + 250)/20 = 18.5.
Frequency to median class	Cumulative counts 3,10,20 reach the halfway position at the second class's upper boundary. With classes [0,10), [10,20), [20,30), interpolation gives 20.

Read the representations

Read what the dataset can support

A group recorded 30 journey times in minutes. Grouping hides the individual times, so midpoint means and interpolated medians are estimates.

Two adjacent equal peak frequencies here would make the single-modal-class formula unsuitable. Use the mean or median asked for, and explain the limitation.

Journey times

Minutes	Frequency
0–10	5
10–20	10
20–30	10
30–40	5

A survey probability requires a selection procedure

Uniformly selecting one of the 30 recorded journey records gives each record equal chance. Selecting a class label uniformly is a different experiment.

A probability for these records does not prove the same probability for all future journeys.

1. **Define the unit**One recorded journey
2. **Define an event**For example: time below 10 minutes
3. **Count within this dataset**5 qualifying records out of 30

Heads	Tosses	Share
3	4	3/4
5	10	1/2

Which trial set landed closer to half heads?

The second. The first did not disprove a fair-coin model.

WORKED EXAMPLES

From the givens to the answer

A separate dataset: 20 parcel masses

1. Classes 0–2,2–4,4–6 kg have frequencies 4,12,4. Midpoints are 1,3,5.
2. Mean estimate=(4+36+20)/20=3 kg. Median class is 2–4; estimate=2+[(10−4)/12]2=3 kg.
3. The symmetry explains agreement here. It does not make the estimated summaries exact raw-data values.

Check another case

Example 1

journey-time data

Minutes	0–10	10–20	20–30	30–40
Frequency	5	10	10	5

Minutes, lower-inclusive upper-exclusive classes. Counts total 30 journeys.

Find P(time at least 10 minutes) for a uniformly selected record.

5/6

25 of the 30 records qualify, or use the complement of 5/30.

Example 2

journey-time data

Minutes	0–10	10–20	20–30	30–40
Frequency	5	10	10	5

Minutes, lower-inclusive upper-exclusive classes. Counts total 30 journeys.

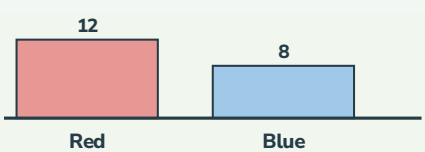
Does this table establish that a future journey has probability 1/6 of taking under 10 minutes?

The 1/6 is a probability for uniform selection from the observed records.

A future-journey claim needs an appropriate sampling and stability model.

Work backwards and test the limits

Problem	Derivation
A register has 5 records in [0,10), 9 in [10,20) and 6 in [20,30). Can you find the exact chance of selecting a record under 20?	Yes, given uniform selection from these 20 records. Fourteen qualify, so the probability is 14/20 = 7/10.
Can you find the exact mean from that register?	Use midpoints 5, 15, 25 to estimate (25 + 135 + 150)/20 = 15.5. Moving observations within their classes can change the true mean.
Which class contains the median?	The cumulative frequencies are 5, 14, 20. The 10th and 11th records both fall in [10,20), so the median class is known even though the individual values are hidden.



Estimate the chance of red from these twenty draws.

12/20 = 3/5. Record whether each counter was replaced before comparing trials.

ILLUSTRATED PROBLEM SHEET

Decide which population a claim describes

A fair sample, a reliable measure and an honest display each answer a different concern.

1

Question	Population
School travel	all pupils
Club travel	club members

Who is missing from a cycling-club survey of school transport?

Pupils outside the club. They may use transport differently.

2

Group	Records	Cycle
A	20	8
B	10	6

Pool the observations. What fraction cycle?

14/30 = 7/15. Averaging 40% and 60% would give both groups equal weight.

3

Day	Monday	Tuesday
Rain?	yes	no
Walkers	8	20

Does the difference prove rain caused the change?

It is consistent with that idea, but other day-to-day differences may explain it.

4

Measure	Before	After
Total pupils	20	40
Cyclists	10	16

Which rose: the headcount or share cycling?

The count rose by six. The share fell from 50% to 40%.

Same method on more volunteers
may retain selection bias.

Sampling across every year group
can improve representation.

CHAPTER PROBLEM

Which is more representative of the whole school: every tenth name after a random start on a mixed pupil list, or the first ten volunteers?

The list method can cover all pupils. Volunteers select themselves and can differ systematically.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Data A

journey-time data

Minutes	0–10	10–20	20–30	30–40
Frequency	5	10	10	5

Minutes, lower-inclusive upper-exclusive classes. Counts total 30 journeys.

Try the problem	Solution and reasoning
01 · Data A Estimate the mean journey time in minutes.	20 Use the class midpoints, 5, 15, 25 and 35 minutes. Their weighted total is 25 + 150 + 250 + 175 = 600 minutes. Dividing by 30 journeys gives an estimated mean of 20 minutes.
02 · Data A How many records have journey time at least 20 minutes?	15 The last two intervals contain 10 and 5 journeys. Both include only times of at least 20 minutes, so the count is 10 + 5 = 15.
03 · Data A Estimate the grouped median journey time in minutes.	20 N/2=15 is the boundary between middle classes. Interpolating either adjacent class gives 20.
04 · Data A Would averaging the class frequencies produce the mean journey time? Explain.	Frequencies count records rather than minutes. Use class midpoints weighted by frequencies, with time units.
05 · Data A Why should the single-modal-class formula not be forced onto these data?	The two adjacent middle classes share the highest frequency. There is no single clear modal class; describe the plateau instead of selecting one arbitrarily.

Set	Hits	Tries
A	3	5
B	9	15

Combine the evidence.

Twelve hits in twenty tries gives 3/5. Pool counts before dividing.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 10 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 10 guides →

Keep exploring

GRADE 10 · TOPIC GUIDE

Counting outcomes without double-counting

Open guide →

GRADE 10 · TOPIC GUIDE

Classical probability and complements

Open guide →

Visit bananalearn.com →