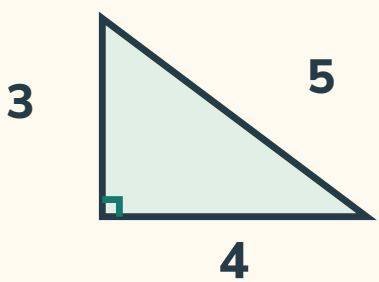
MATHS FIELD GUIDE · GRADE 10

Reciprocal trigonometric ratios

OPENING PROBLEM

For the acute angle opposite 3, find sine and cosine.



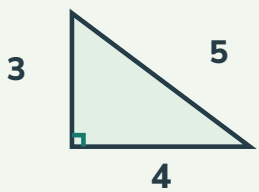
02 Choose the sides for this angle
03 Rules and worked cases
04 From the givens to the answer
05 Check the domain before dividing
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Choose the sides for this angle

In a right triangle the hypotenuse stays fixed. Opposite and adjacent swap when the other acute angle is chosen.

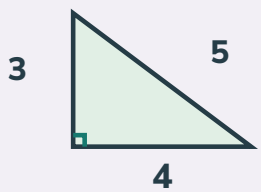
1



For the acute angle opposite 3, find sine and cosine.

sin θ = 3/5 and cos θ = 4/5. The hypotenuse is the denominator in each.

2



Use the angle opposite 4 instead. What happens to tangent?

It becomes 4/3. For the other angle it was 3/4.

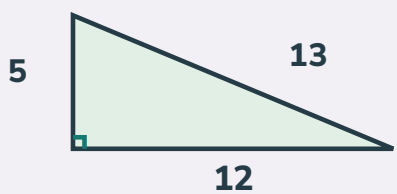
3

$$\sin \theta = 5/13$$
$$\downarrow$$
$$\text{cosec } \theta = 13/5$$

Why does the fraction invert?

Cosecant is hypotenuse over opposite. Its product with sine is one wherever both are defined.

4



Use the triangle to check $\sin^2 \theta + \cos^2 \theta$.

The numerator is 25 + 144 = 169 over 169. The sum is one.

Opposite 5, adjacent 12
tan θ = 5/12.

Same triangle, other acute angle
tan = 12/5.

CHAPTER PROBLEM

For acute θ with sin θ=8/17, find cosec θ.

17/8. Invert the nonzero sine.

MATHS REFERENCE · GRADE 10

Reciprocal trigonometric ratios

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
cosec θ = 1/sin θ	With sin θ = 3/5, cosec θ = 5/3.
sec θ = 1/cos θ	In a 3–4–5 triangle with adjacent 4, sec θ = 5/4.
cot θ = 1/tan θ	If opposite 3, adjacent 4, then cot θ = 4/3. A zero denominator makes a ratio undefined.

Read the representations

Three reciprocal ratios add three names

For an acute angle in a right triangle, sec θ = hypotenuse/adjacent = 1/cos θ; cosec θ = hypotenuse/opposite = 1/sin θ.

cot θ = adjacent/opposite = cos θ/sin θ. The reciprocal 1/tan θ is valid only where tan θ is itself defined and nonzero.

A denominator must not be zero. These six right-triangle ratios are positive for acute angles.

Six ratios for a 3–4–5 triangle, θ opposite 3

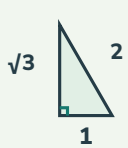
Basic ratio	Value	Reciprocal	Value
sin θ	3/5	cosec θ	5/3
cos θ	4/5	sec θ	5/4
tan θ	3/4	cot θ	4/3

Match a name to its side ratio

Reciprocal means invert a nonzero ratio; it does not mean subtract it from 1.

Ratios do not carry centimetres or metres. The side units cancel after conversion.

The names sine and cosecant pair together, while cosine and secant pair together.



Which acute angle faces the short leg?

30°. Thus sin 30° = 1/2 and cos 30° = √3/2.

WORKED EXAMPLES

From the givens to the answer

Find secant from a scaled triangle

1. For acute θ, adjacent=20 and hypotenuse=29, with opposite=21.
2. cos θ=20/29, so sec θ=29/20.
3. cosec θ=29/21 and cot θ=20/21.

Recover a basic ratio

1. Given cosec θ=17/8, sin θ=8/17.
2. For an acute θ, cos θ=15/17 by a matching 8–15–17 triangle.
3. Then tan θ=8/15 and cot θ=15/8.

Does cosec θ mean 1 – sin θ?

1. Cosecant is the multiplicative reciprocal of sine.
2. Their product is 1 when both exist.
3. Subtracting from 1 gives a different quantity.

Check another case

Example 1

For acute θ with cos θ=48/50, find sec θ.

50/48

Invert the nonzero cosine.

Example 2

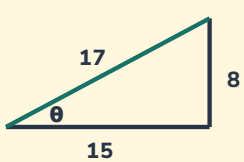
For tan θ=15/36, which is cot θ?

36/15 · 15/36 · 39/36 · 15/39

36/15

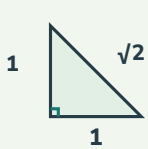
Invert the tangent for this acute angle.

Choose the correct reciprocal



A right triangle has legs 8 and 15 and hypotenuse 17. For the acute angle opposite 8, sin θ = 8/17 and cosec θ = 17/8.

Sec θ uses the adjacent side, so it is 17/15. Multiplying each ratio by its own reciprocal gives one.



Why is tan 45° exactly one?

The legs match. Sine and cosine each equal 1/√2.

ILLUSTRATED PROBLEM SHEET

Check the domain before dividing

Begin with $\sin^2\theta + \cos^2\theta = 1$. Dividing requires a nonzero divisor.

1

$$\sin^2\theta + \cos^2\theta = 1$$
$$\downarrow$$
divide by $\cos^2\theta$
$$\downarrow$$
$$\tan^2\theta + 1 = \sec^2\theta$$

Which angles are excluded from this division?

Angles where cosine equals zero, including 90°.

2

$$\sin\theta = 3/5$$
$$\downarrow$$
$$\cos^2\theta = 16/25$$
$$\downarrow$$
$$\cos\theta = 4/5 \text{ if acute}$$

Where does the acute-angle condition enter?

When choosing the positive square root. The squared relation alone does not determine the sign.

3

$$1 - \cos^2\theta$$
$$\downarrow$$
$$(1 - \cos\theta)(1 + \cos\theta)$$

Simplify $(1 - \cos^2\theta)/(1 - \cos\theta)$.

It equals $1 + \cos\theta$ wherever $\cos\theta \neq 1$. Keep the original restriction.

4

$$(\sin\theta + \cos\theta)^2$$
$$\downarrow$$
$$1 + 2\sin\theta\cos\theta$$

Is the square always one?

At 45° the cross term equals one, giving two. The proposed identity fails.

$\tan^2\theta + 1 = \sec^2\theta$ requires $\cos\theta \neq 0$.

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ requires $\sin\theta \neq 0$.

CHAPTER PROBLEM

For acute θ with $\cos\theta = 24/25$, find $\sec\theta$.

25/24. Invert the nonzero cosine.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Could 7/5 be the sine of an acute angle?	No. Its opposite side would exceed the hypotenuse. A reciprocal ratio such as cosecant can exceed one.
02 If sec θ + tan θ = 4, find sec θ − tan θ.	Their product is $\sec^2\theta - \tan^2\theta = 1$. The second factor is 1/4.
03 For tan θ=16/30, which is cot θ? A) 30/16 B) 16/30 C) 34/30 D) 16/34	30/16 Invert the tangent for this acute angle.
04 Given sin θ=14/50, which statement about cosec θ is correct? A) sin θ × cosec θ = 1 B) sin θ + cosec θ = 1 C) cosec θ = 1 − sin θ D) cosec θ is negative	sin θ × cosec θ = 1 Reciprocal quantities have product 1.
05 For adjacent 36 and hypotenuse 39, evaluate cos θ × sec θ.	1 The two nonzero ratios are reciprocals.
06 A matching triangle has opposite 10 and adjacent 24. Find cot θ.	12/5 Divide adjacent by opposite and simplify.

Can the secant of an acute angle be 0.8?

For an acute angle the cosine lies strictly between zero and one. Its reciprocal therefore exceeds one.

If sec θ = 5/4, then cos θ = 4/5 is possible. A 3–4–5 triangle supplies a check.

θ	0°	90°
cos θ	1	0
sin θ	0	1

At which endpoint does tan θ fail?

At 90°. Cosine is zero, so the quotient sine/cosine is undefined.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why sin θ=8/17 gives cosec θ greater than 1.	The opposite leg is shorter than the hypotenuse. Sine lies between 0 and 1 for an acute angle. Its positive reciprocal exceeds 1.
08 In a right triangle with legs 7,24, identify the denominator condition for cot θ.	cot θ is adjacent divided by opposite. The opposite must be nonzero. The acute-angle non-degenerate triangle satisfies that requirement.
09 For acute θ with sin θ=7/25, find cosec θ.	25/7 Invert the nonzero sine.
10 For acute θ with cos θ=24/26, find sec θ.	26/24 Take the reciprocal of the cosine. The secant is 26/24, or 13/12.
11 For acute θ with sin θ=16/34, find cosec θ.	34/16 Invert the nonzero sine.

$\theta = 60°$
↓
$\sin^2\theta = 3/4$
↓
$\cos^2\theta = 1/4$

Check the identity at an exact angle.

Three quarters plus one quarter equals one. The individual ratios are square roots of these values.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 10 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 10 guides →

Keep exploring

GRADE 10 · TOPIC GUIDE

Right-triangle ratios

Open guide →

GRADE 10 · TOPIC GUIDE

Exact values at 30, 45 and 60 degrees

Open guide →

Visit bananalearn.com →