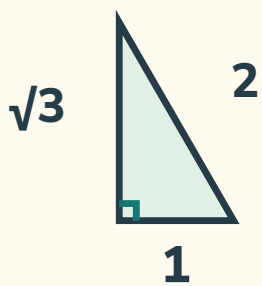
MATHS FIELD GUIDE · GRADE 10

Ratios at zero and ninety degrees

OPENING PROBLEM

Which acute angle faces the short leg?



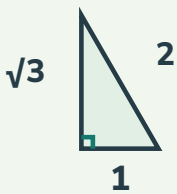
02 Rebuild the exact trigonometric values
03 Rules and worked cases
04 From the givens to the answer
05 Check the domain before dividing
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Rebuild the exact trigonometric values

Use half an equilateral triangle or half a square. At 0° and 90° use unit-circle coordinates.

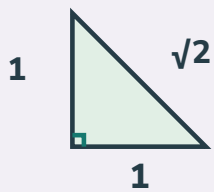
1



Which acute angle faces the short leg?

30°. Thus sin 30° = 1/2 and cos 30° = √3/2.

2



Why is tan 45° exactly one?

The legs match. Sine and cosine each equal 1/√2.

3

θ	0°	90°
cos θ	1	0
sin θ	0	1

At which endpoint does tan θ fail?

At 90°. Cosine is zero, so the quotient sine/cosine is undefined.

4

$$\theta = 60°$$
$$\downarrow$$
$$\sin^2\theta = 3/4$$
$$\downarrow$$
$$\cos^2\theta = 1/4$$

Check the identity at an exact angle.

Three quarters plus one quarter equals one. The individual ratios are square roots of these values.

sin 30° equals cos 60°
both 1/2.

sin 60° equals cos 30°
both √3/2.

CHAPTER PROBLEM

Evaluate 4sin0°.

0. The endpoint sine is zero.

MATHS REFERENCE · GRADE 10

Ratios at zero and ninety degrees

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
At 0°	On the unit circle the point is (1,0), so cos 0° = 1, sin 0° = 0 and tan 0° = 0.
At 90°	The point is (0,1). Cos 90° = 0 and sin 90° = 1. Tan 90° is undefined.
Reciprocals	sec 0° = 1 and cosec 90° = 1. Cosec 0° and sec 90° divide by zero.

Read the representations

Follow sine and cosine to the endpoints

In a unit-circle first-quadrant model, the point at angle θ has horizontal coordinate cos θ and vertical coordinate sin θ.

At 0°, the point is (1,0), so cos0°=1 and sin0°=0. At 90°, it is (0,1), so cos90°=0 and sin90°=1.

These endpoint values extend the acute-triangle ratios by limiting position; a collapsed triangle itself is not a non-degenerate right triangle.

Endpoint values from coordinates

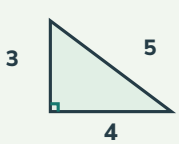
θ	cos θ	sin θ	tan θ
0°	1	0	0
90°	0	1	undefined

Check each quotient's own denominator

tan θ=sin θ/cos θ and sec θ=1/cos θ are undefined at 90°.

cot θ=cos θ/sin θ and cosec θ=1/sin θ are undefined at 0°. At 90°, cot θ=0 and cosec θ=1.

Do not use cot θ=1/tan θ at an angle where tan θ is undefined. Start from cos θ/sin θ instead.



For the acute angle opposite 3, find sine and cosine.

sin θ = 3/5 and cos θ = 4/5. The hypotenuse is the denominator in each.

WORKED EXAMPLES

From the givens to the answer

Evaluate an endpoint expression

1. $3\cos0°+2\sin90° = 3(1)+2(1)=5$.
2. Each term uses a defined endpoint value.

Identify an undefined ratio

1. $\sec90°$ would require $1/\cos90°=1/0$.
2. There is no real value for this quotient. It is not zero.

Is $\tan90°$ zero because cosine is zero?

1. Cosine is the denominator of tangent.
2. $\sin90°/\cos90°$ would divide 1 by 0.
3. The quotient is undefined, while $\cot90°=0/1$ is defined.

Check another case

Example 1

Evaluate $8\cos0°$.

8

The endpoint cosine is one.

Example 2

Which is the value of $9\tan90°$?

Undefined · 0 · 9 · 1

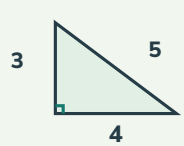
Undefined

The tangent denominator is zero.

Read the quarter-turn endpoints

On a unit circle, the point at 0° is (1, 0) and at 90° is (0, 1). Cosine gives the x-coordinate, sine the y-coordinate.

At 0°, $\tan\theta = 0/1 = 0$. At 90°, the denominator vanishes, so the tangent is undefined.



Use the angle opposite 4 instead. What happens to tangent?

It becomes 4/3. For the other angle it was 3/4.

ILLUSTRATED PROBLEM SHEET

Check the domain before dividing

Begin with $\sin^2\theta + \cos^2\theta = 1$. Dividing requires a nonzero divisor.

1

$$\sin^2\theta + \cos^2\theta = 1$$
$$\downarrow$$
divide by $\cos^2\theta$
$$\downarrow$$
$$\tan^2\theta + 1 = \sec^2\theta$$

Which angles are excluded from this division?

Angles where cosine equals zero, including 90°.

2

$$\sin\theta = 3/5$$
$$\downarrow$$
$$\cos^2\theta = 16/25$$
$$\downarrow$$
$$\cos\theta = 4/5 \text{ if acute}$$

Where does the acute-angle condition enter?

When choosing the positive square root. The squared relation alone does not determine the sign.

3

$$1 - \cos^2\theta$$
$$\downarrow$$
$$(1 - \cos\theta)(1 + \cos\theta)$$

Simplify $(1 - \cos^2\theta)/(1 - \cos\theta)$.

It equals $1 + \cos\theta$ wherever $\cos\theta \neq 1$. Keep the original restriction.

4

$$(\sin\theta + \cos\theta)^2$$
$$\downarrow$$
$$1 + 2\sin\theta\cos\theta$$

Is the square always one?

At 45° the cross term equals one, giving two. The proposed identity fails.

$\tan^2\theta + 1 = \sec^2\theta$ requires $\cos\theta \neq 0$.

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ requires $\sin\theta \neq 0$.

CHAPTER PROBLEM

Evaluate $5\cos 0°$.

5. The endpoint cosine is one.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is cosec 0° equal to zero?	No. It is 1/sin 0°, so the denominator is zero and it is undefined.
02 If sec θ + tan θ = 4, find sec θ − tan θ.	Their product is $\sec^2\theta - \tan^2\theta = 1$. The second factor is 1/4.
03 Which is the value of 7tan90°? A) Undefined B) 0 C) 7 D) 1	Undefined The tangent denominator is zero.
04 Which is the value of 8cot90°? A) 0 B) Undefined C) 8 D) 1	0 cot90°=cos90°/sin90°=0.
05 Evaluate 9sin90° + cos90°.	9 Use sine one and cosine zero.
06 Evaluate 3cosec90°.	3 cosec90°=1/sin90°=1.

Which endpoint makes the reciprocal fail?

The cosecant is 1/sin θ. At 0° that denominator is zero. At 90° it equals one, so cosec 90° = 1.

Secant reverses these endpoint roles because it is 1/cos θ. It is defined at 0° and undefined at 90°.

sin θ = 5/13
↓
cosec θ = 13/5

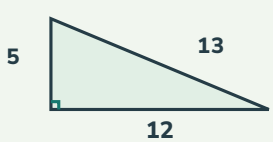
Why does the fraction invert?

Cosecant is hypotenuse over opposite. Its product with sine is one wherever both are defined.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why 4sec90° has no real value.	cos90°=0. Secant would take its reciprocal. Division by zero remains undefined after multiplication by 4.
08 Explain why 5cot90° can be defined even though tan90° is undefined.	Use cotθ=cosθ/sinθ directly. At 90 degrees the denominator is 1. The result is zero; a reciprocal of an undefined expression was not used.
09 Evaluate 5sin0°.	0 The endpoint sine is zero.
10 Evaluate 6cos0°.	6 The endpoint cosine is one.
11 Evaluate 7sin0°.	0 The endpoint sine is zero.



Use the triangle to check sin² θ + cos² θ.

The numerator is 25 + 144 = 169 over 169. The sum is one.

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