MATHS FIELD GUIDE · GRADE 10

Quadratics and zero-product reasoning

OPENING PROBLEM

Which products make the middle term?

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

02 Choose a quadratic method
03 Rules and worked cases
04 From the givens to the answer
05 Connect factors, roots and the constant term
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
$x = \pm 3$.

$x^2 + 9 = 0$
no real roots.

CHAPTER PROBLEM

For $x^2-11x+28=0$, find the smaller root.

4. Factor as $(x-4)(x-7)=0$. The smaller root is 4.

MATHS REFERENCE · GRADE 10

Quadratics and zero-product reasoning

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Set equal to zero	$x^2 = 5x$ gives $x(x - 5) = 0$. Roots are 0 and 5.
One repeated root	$(x + 4)^2 = 0$ forces $x = -4$ twice as a factor.
Nonzero product	$(x - 2)(x + 1) = 4$ does not imply either factor is zero. Expand and move four to the left first.

Read the representations

Factor pairs reverse multiplication

For x^2+bx+c, seek integers p,q with $p+q=b$ and $pq=c$. Then $(x+p)(x+q)=x^2+(p+q)x+pq$.

To solve an equation, first make one side zero. If a product is zero, at least one factor is zero; this does not hold for a product equal to a nonzero number.

Track both checks

Expression	Sum	Product	Factors
$x^2+7x+12$	$3+4=7$	$3\times4=12$	$(x+3)(x+4)$
x^2-x-6	$2+(-3)=-1$	$2\times(-3)=-6$	$(x+2)(x-3)$

Factors and solutions have different signs

From $(x-2)(x+5)=0$, the roots are 2 and -5. Check them in the original equation.

Not every quadratic factors into integer linear factors; failing to find integer factors is not evidence that no real roots exist.

$$(x^2 - 9)/(x - 3)$$
$$\downarrow$$
$$(x - 3)(x + 3)/(x - 3)$$
$$\downarrow$$
$$x + 3 \text{ for } x \neq 3$$

Which input is still forbidden?

Three. The original expression has denominator zero there.

WORKED EXAMPLES

From the givens to the answer

Solve $x^2-5x+6=0$

1. Choose -2 and -3: they sum to -5 and multiply to 6.
2. Factor $(x-2)(x-3)=0$. Roots are 2 and 3.
3. Substitute to check each root.

Bring all terms to one side

1. For $x^2+4x=12$, rewrite $x^2+4x-12=0$.
2. Factor $(x+6)(x-2)=0$ to obtain $x=-6$ or $x=2$.

Can $x^2=5x$ be solved by dividing by x only?

1. Division by x discards the possible root $x=0$.
2. Factor $x(x-5)=0$ to retain both roots.

Check another case

Example 1

For $x(x-8)=0$, find the nonzero root.

8

Apply the zero-product rule.

Example 2

Which factors $x^2+21x+108$?

$(x+9)(x+12)$ · $(x-9)(x-12)$ · $(x+9)(x-12)$ · $x(x+21)$

$(x+9)(x+12)$

Check both sum and product.

What could go wrong when cancelling x?

For $x^2 = 5x$, first bring everything to one side: $x(x - 5) = 0$. Either $x = 0$ or $x = 5$.

Dividing by x at the start would discard zero. Check both values in the original equation.

x	2	3	4
Original	5	undefined	7
Simplified	5	6	7

Where do the rows differ?

At 3 the simplified formula alone fills a hole that the original expression has.

ILLUSTRATED PROBLEM SHEET

Connect factors, roots and the constant term

A product is zero when at least one factor is zero.

1

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

2

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

3

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

4

$$\text{Sum} = -1$$
$$\downarrow$$
$$\text{Product} = -6$$
$$\downarrow$$
$$x^2 + x - 6$$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are (x − 2) and (x + 3).

$p(x) = (x - 4)^2$
one distinct zero.

$p(x) = (x - 4)(x + 4)$
two distinct zeros.

CHAPTER PROBLEM

For x(x−5)=0, find the nonzero root.

5. Apply the zero-product rule.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 What happens to $x^2 - 6x + k = 0$ at $k = 9$?	It becomes $(x - 3)^2 = 0$. The two root values coincide at 3.
02 Can a nonzero quadratic have three distinct real zeros?	No. Three distinct linear factors would require degree at least three.
03 Which factors $x^2+17x+70$? A) $(x+7)(x+10)$ B) $(x-7)(x-10)$ C) $(x+7)(x-10)$ D) $x(x+17)$	$(x+7)(x+10)$ Check both sum and product.
04 Why can $(x-8)(x+11)=6$ not be solved by setting each factor to zero? A) The product is not zero B) The factors have unlike signs C) All quadratics have one root D) A variable cannot be factored	The product is not zero The zero-product rule needs a zero product.
05 The roots of $(x-9)(x+12)=0$ include 9. Find the other root.	-12 The first factor gives 9. Setting $x + 12$ equal to zero gives -12, which makes the entire product zero.
06 For $x^2-9=0$, find its positive root.	3 Use the difference of two squares.

$$\frac{x + 2}{x + 5}$$
$$\downarrow$$
$$x \neq -5$$

Can x be cancelled from this fraction?

No. It is part of two sums, not a common factor.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why $x^2=4x$ has two distinct roots.	Move all terms to one side. Factor $x(x-4)=0$. The roots are zero and 4; dividing by x would lose zero.
08 Check the factorisation $x^2+13x+40=(x+5)(x+8)$.	Multiply both pairs of terms. The middle coefficient is the sum 13. The constant is the product 40.
09 For $x^2-13x+40=0$, find the smaller root.	5 Factor as $(x-5)(x-8)=0$. The smaller root is 5.
10 For $x(x-6)=0$, find the nonzero root.	6 Apply the zero-product rule.
11 For $x^2-17x+70=0$, find the smaller root.	7 Factor as $(x-7)(x-10)=0$. The smaller root is 7.

$$\frac{2x}{4x^2} \rightarrow \frac{1}{2x} \rightarrow x \neq 0$$

Check the cancellation at x = 2.

Both forms give 1/4. At zero neither is defined.

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