MATHS FIELD GUIDE · GRADE 10

Quadratic models and feasible answers

OPENING PROBLEM

Which width fits?

$$w(w + 2) = 35$$
$$\downarrow$$
$$(w - 5)(w + 7) = 0$$
$$\downarrow$$
$$w = 5 \text{ or } -7$$

02 Keep only roots that fit the situation
03 Rules and worked cases
04 From the givens to the answer
05 Choose a quadratic method
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Keep only roots that fit the situation

Generate algebraic candidates first. Check the domain and the original conditions for each one.

1

$$w(w + 2) = 35$$
$$\downarrow$$
$$(w - 5)(w + 7) = 0$$
$$\downarrow$$
$$w = 5 \text{ or } -7$$

Which width fits?

5. Then the length is 7 and the area is 35. A negative width fails the model.

2

$$n(n + 1) = 42$$
$$\downarrow$$
$$(n - 6)(n + 7) = 0$$

Must a root always be rejected?

For any integers, both pairs (6, 7) and (–7, –6) work. “Positive integers” keeps only the first.

3

$$t(8 - t) = 12$$
$$\downarrow$$
$$(t - 2)(t - 6) = 0$$

If t is a time between 0 and 8, which roots survive?

Both 2 and 6 lie in the allowed interval. Both give the same product.

4

$$x^2 = x$$
$$\downarrow$$
$$x(x - 1) = 0$$
$$\downarrow$$
$$x = 0 \text{ or } 1$$

What happens if x represents the number of extra seats?

Zero may be valid. The domain “whole number” includes zero.

A squared length equals 16
physical length 4.

A signed coordinate squared equals 16
coordinate 4 or –4.

CHAPTER PROBLEM

A positive rectangle has width x, length x+2 and area 35. Find its width.

5. $x(x+2)=35$ becomes $(x-5)(x+7)=0$. Reject the negative width and use $x=5$.

MATHS REFERENCE · GRADE 10

Quadratic models and feasible answers

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Build from area	A rectangle has width x and length x + 3 with area 40. Equation $x^2 + 3x - 40 = 0$.
Keep feasible roots	$(x - 5)(x + 8) = 0$ gives 5 and −8. Width must be positive, so dimensions are 5 by 8.
Check what was asked	If the question asks the boundary length, continue to P = 2(5 + 8) = 26 units.

Read the representations

Translate quantities before solving a quadratic

Define the unknown with units and its physical restrictions. Products of two changing lengths or other quantities can create a quadratic equation.

For a rectangle of width x and length x+3 with area 40, x(x+3)=40. Algebraic roots need checking against positive dimensions.

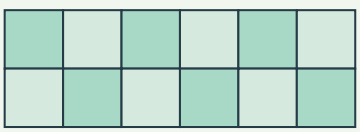
The model carries a domain

Quantity	Expression	Condition
Width	x	x>0
Length	x+3	x+3>0
Area	x(x+3)	40 square units

A mathematical root can fail the context

Solving $x^2+3x-40=(x+8)(x-5)=0$ gives x=−8 or 5. Only x=5 gives positive lengths.

Check the surviving dimensions in the original area statement and report a labelled answer. Discarding an impossible root requires its contextual reason.



Twelve square tiles in two rows. What are the measurements?

Area 12 square units. Boundary 6 + 2 + 6 + 2 = 16 units.

WORKED EXAMPLES

From the givens to the answer

Consecutive positive integers

1. If their product is 56, define $n(n+1)=56$ with positive integer n.
2. Factor $(n+8)(n-7)=0$. Only $n=7$ fits the domain, so the numbers are 7 and 8.

A model may have no physical solution

1. A square with side x would need $x^2=-9$ for a negative stated area.
2. No real positive side exists; inspect the inconsistent givens rather than inventing a dimension.

Must every quadratic root be reported as a physical answer?

1. First check units, sign, integrality and original conditions.
2. A root can solve the equation while failing the model's domain.

Check another case

Example 1

Two consecutive positive integers have product 90. Find the smaller.

9

Use $n(n+1)$=the given product and positivity.

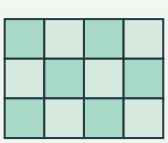
Example 2

An area model gives roots 10 and -12 for width. Which is admissible?

10 · -12 · Both · Neither because all roots are invalid

10

Physical width must be positive.



Rearrange the same twelve tiles into three rows. What changes?

Area stays 12. Perimeter falls to 14 because more tile edges meet inside.

ILLUSTRATED PROBLEM SHEET

Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
$x = \pm 3$.

$x^2 + 9 = 0$
no real roots.

CHAPTER PROBLEM

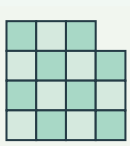
Two consecutive positive integers have product 42. Find the smaller.

6. Use n(n+1)=the given product and positivity.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A rectangle has width w and length w + 1. Could w = −4 be accepted because the product is positive?	No. Both dimensions must be positive separately.
02 What happens to $x^2 - 6x + k = 0$ at k = 9?	It becomes $(x - 3)^2 = 0$. The two root values coincide at 3.
03 An area model gives roots 8 and -10 for width. Which is admissible? A) 8 B) -10 C) Both D) Neither because all roots are invalid	8 Physical width must be positive.
04 A model of whole objects gives a positive root 9.5. What should be checked? A) Whether the count domain permits a noninteger B) Only the font size C) Whether all decimals are irrational D) Nothing; every positive root works	Whether the count domain permits a noninteger Counts of indivisible objects require integers.
05 A square has exact area 100. Find its positive side.	10 Take the nonnegative root and use positive area.
06 A rectangle has sides 4 and 6. Find its perimeter.	20 Report a length check after solving.



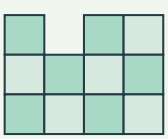
Remove the corner tile from a four-by-four square. What changes?

Area falls from 16 to 15. Two lost outer edges are replaced by two exposed ones, so perimeter stays 16.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Build and justify an equation for a positive width x and length x+2 with area 35.	Define x and the length unit. Use x(x+2)=35 with x>0. Solve, reject the negative root, and check the original area.
08 Explain why roots 6 and -8 require a domain decision rather than choosing the first root arbitrarily.	Both may solve the polynomial equation. Only positive dimensions fit this stated rectangle. Select by conditions and verify the original model.
09 A positive rectangle has width x, length x+2 and area 48. Find its width.	6 x(x+2)=48 becomes (x−6)(x+8)=0. Reject the negative width and use x=6.
10 Two consecutive positive integers have product 56. Find the smaller.	7 Use n(n+1)=the given product and positivity.
11 A positive rectangle has width x, length x+2 and area 80. Find its width.	8 x(x+2)=80 becomes (x−8)(x+10)=0. Reject the negative width and use x=8.



Remove a top tile away from a corner. What changes?

Area falls from 12 to 11. One outer edge disappears and three appear, so perimeter rises from 14 to 16.

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