MATHS FIELD GUIDE · GRADE 10

Prime structure and unique factorisation

OPENING PROBLEM

Read the HCF from the exponents.

Number	Power of 2	Power of 3
24	3	1
36	2	2

02 Use the prime powers as building blocks
03 Rules and worked cases
04 From the givens to the answer
05 Divisibility without listing every multiple
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Use the prime powers as building blocks

Prime exponents record how many copies of each prime a number contains.

1

$$72 = 8 \times 9$$
$$\downarrow$$
$$2^3 \times 3^2$$

Why do two different factor trees end alike?

Their terminal prime factors, counted with multiplicity, are unique apart from order.

2

Number	Power of 2	Power of 3
24	3	1
36	2	2

Read the HCF from the exponents.

Take the smaller exponent in each column. HCF = $2^2 \times 3 = 12$.

3

Number	Power of 2	Power of 3
24	3	1
36	2	2

What is the first shared multiple?

Take the larger exponents. LCM = $2^3 \times 3^2 = 72$.

4

$$108 = 2^2 \times 3^3$$
$$\downarrow$$
$$108 \times 3 = 2^2 \times 3^4$$

Which smallest positive integer makes 108 a square when multiplied by it?

Three supplies the missing partner for the odd exponent. The result is 18^2.

$180 = 2^2 \times 3^2 \times 5$
multiply by 5 for a square.

$180 \div 5 = 36$
a square already.

CHAPTER PROBLEM

Find the exponent of 2 in the prime factorisation of 96.

5. $96 = 2^5 \times 3$.

MATHS REFERENCE · GRADE 10

Prime structure and unique factorisation

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Recognise a prime power	$108 = 4 \times 27 = 2^2 \times 3^3$. Any full prime factor tree gives these exponents.
HCF versus LCM	For $12 = 2^2 \times 3$ and $18 = 2 \times 3^2$, take smaller powers for 6 and larger powers for 36.
Perfect square test	$180 = 2^2 \times 3^2 \times 5$. Multiply by 5 to make each exponent even: $900 = 30^2$.

Read the representations

Prime factors are the building blocks

A prime is an integer greater than 1 with exactly two positive factors. 1 is not prime.

Every integer greater than 1 is a product of primes, unique apart from their order. Different factor trees must end with the same prime factors.



$60 = 2^2 \times 3 \times 5$

HCF keeps shared factors; LCM covers both numbers

For positive integers, HCF uses the smaller exponent of each prime. LCM uses the larger exponent.

A missing prime has exponent zero. HCF divides both numbers; both numbers divide the LCM.

Compare 72 and 120

Prime	72	120	HCF / LCM
2	3 copies	3 copies	3 / 3 copies
3	2 copies	1 copy	1 / 2 copies
5	0 copies	1 copy	0 / 1 copies

$\sqrt{2} = p/q$
↓
$p^2 = 2q^2$
↓
p is even

Why must p be even?

An odd integer has an odd square. An even square can only come from an even integer.

WORKED EXAMPLES

From the givens to the answer

Find HCF and LCM of 18 and 30

1. $18 = 2\times3^2$; $30 = 2\times3\times5$.
2. HCF $= 2\times3 = 6$. LCM $= 2\times3^2\times5 = 90$.
3. Check for this pair: $6\times90 = 18\times30 = 540$. The identity applies to two positive integers.

Can a product have a new prime divisor?

1. $84\times25 = (2^2\times3\times7)\times5^2$.
2. Its prime divisors are exactly 2, 3, 5 and 7.
3. 11 cannot divide this product: unique prime factorisation leaves no factor 11.

Check another case

Example 1

Explain why $7\times11\times13 + 13$ is composite.

Factor it as $13(7\times11 + 1) = 13\times78$.

Both factors exceed 1, so it is composite.

Example 2

Find HCF(35, 64).

1

35 = 5×7 and $64 = 2^6$ have no common prime factor.

Work backwards and test the limits

Problem	Derivation
Which smallest positive integer must divide 540 to leave a perfect square?	$540 = 2^2 \times 3^3 \times 5$. Remove one 3 and one 5 to leave $2^2 \times 3^2 = 36$. The smallest divisor is 15.
Can 18 divide $2^3 \times 3 \times 5$?	18 needs two factors of 3. The product contains just one, so 18 cannot divide it, even though 2 and 3 both do.
How many positive factors does 72 have?	$72 = 2^3 \times 3^2$. A divisor chooses an exponent 0–3 for 2 and 0–2 for 3. Four choices times three give twelve factors.

$p = 2k$
↓
$4k^2 = 2q^2$
↓
$q^2 = 2k^2$

What does this imply about q?

It is even too. Both p and q divisible by 2 contradict lowest terms.

ILLUSTRATED PROBLEM SHEET

Divisibility without listing every multiple

A remainder, a common divisor and a repeated interval answer different questions.

1

$$84 = 2 \times 30 + 24$$
$$\downarrow$$
$$30 = 1 \times 24 + 6$$
$$\downarrow$$
$$24 = 4 \times 6$$

Find HCF(84, 30) by remainders.

The last nonzero remainder is 6. Each division preserves the common divisors.

2

Factor	84	30
2	2 copies	1 copy
3	1 copy	1 copy
Other	7	5

Check the HCF and LCM with prime factors.

HCF = 2 × 3 = 6. LCM = 4 × 3 × 5 × 7 = 420.

3

Two bells: 12 and 18 min
↓
LCM = 36 min

If both ring at 9:00, when next together?

9:36. A shared future time needs a multiple of both intervals.

4

30 red and 42 blue
↓
Identical packs
↓
HCF = 6 packs

How many of each colour in each largest-count pack arrangement?

Five red and seven blue. Every pack has the same contents with none left over.

HCF(18,24)
6.

LCM(18,24)
72; their product is 18 × 24.

CHAPTER PROBLEM

Find HCF(48, 80).

16. Both numbers contain the factor 16. Dividing them by it leaves 3 and 5, which have no further common factor.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why is 1 excluded from the primes?	It has only one positive divisor. Allowing extra factors of 1 would also make prime-factor lists non-unique.
02 Can the LCM of two positive integers equal the larger number?	Yes, when the smaller divides the larger. For 6 and 24 it is 24.
03 Find LCM(24, 90).	360 The greatest common factor is 6. The least common multiple is (24 × 90) ÷ 6 = 360.
04 Which statement about 1 is correct? A) It is prime. B) It is neither prime nor composite. C) It has two positive factors. D) It is the smallest composite number.	It is neither prime nor composite. 1 has exactly one positive factor.
05 Two positive integers have product 2160 and HCF 12. Find their LCM.	180 HCF×LCM = product, so LCM = 2160/12 = 180.
06 Why do 60 = 6×10 and 60 = 4×15 not contradict unique prime factorisation?	6, 10, 4 and 15 are not all primes. Both decompositions give $2^2 \times 3 \times 5$ after complete factorisation.

$r = 3 + \sqrt{2}$
↓
$\sqrt{2} = r - 3$

Could r be rational?

If so, subtracting the rational number 3 would make √2 rational. That contradicts the proof.

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