MATHS FIELD GUIDE · GRADE 10

Polynomials and zeros

OPENING PROBLEM

Which table entry is the vertical intercept?

x	−3	0	2
p(x)	0	−6	0

02 Connect factors, roots and the constant term
03 Rules and worked cases
04 From the givens to the answer
05 Choose a quadratic method
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Connect factors, roots and the constant term

A product is zero when at least one factor is zero.

1

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

2

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

3

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

4

$$\text{Sum} = -1$$
$$\downarrow$$
$$\text{Product} = -6$$
$$\downarrow$$
$$x^2 + x - 6$$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are (x − 2) and (x + 3).

$p(x) = (x - 4)^2$
one distinct zero.

$p(x) = (x - 4)(x + 4)$
two distinct zeros.

CHAPTER PROBLEM

Find p(3) when $p(x)=x^2-9$.

0. Substitute 3 for x. Since $3^2 - 9 = 0$, 3 is a zero of this polynomial.

MATHS REFERENCE · GRADE 10

Polynomials and zeros

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Evaluate a polynomial	For $p(x) = x^2 - x - 6$, $p(-2) = 4 + 2 - 6 = 0$.
Read factors as zeros	$(x + 2)(x - 3)$ vanishes at -2 and 3; either factor can be zero.
Count distinct zeros	$(x - 4)^2$ touches the axis at $x = 4$. The zero occurs twice as a factor but gives one intercept.

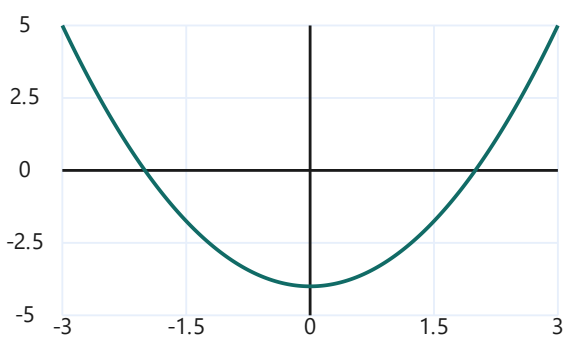
Read the representations

A zero is an input that makes the output zero

A polynomial in x uses nonnegative integer powers of x with real coefficients. After combining terms, the largest power with a nonzero coefficient is its degree.

A zero α satisfies $p(\alpha)=0$. On $y=p(x)$, zeros are the x-coordinates of points on the x-axis. Crossing and touching both count.

A parabola crosses at −2 and 2



$y=x^2-4$ equals zero where the graph meets $y=0$.

Degree limits the number of distinct zeros

A nonzero degree-n polynomial has at most n distinct real zeros. A quadratic can have two, one or no real zeros.

The zero polynomial is a separate case: every input gives zero and its degree is not defined. A nonzero constant has no zeros.

Read the expression first

Expression	Polynomial?	Degree
$3x^2-x+2$	Yes	2
$1/x$	No: x^{-1}	Not applicable
$\sqrt{x}$	No: $x^{1/2}$	Not applicable

x	−1	0	1
y = 2x + 1	−1	1	3

What stays constant?

Each x-step of one raises y by two. The value at zero is one.

WORKED EXAMPLES

From the givens to the answer

Read both zeros from a factorisation

1. $p(x)=(x-1)(x+4)$. At $x=1$ or $x=-4$, one factor is zero.
2. Thus the zeros are 1 and -4. The x-intercepts are $(1,0)$ and $(-4,0)$.
3. The constant $p(0)=-4$ is the y-intercept, not a zero.

Does every quadratic have two distinct real zeros?

1. x^2+1 never equals zero for a real input.
2. A degree gives an upper bound; it does not guarantee that many real zeros.

Check another case

Example 1

Find the negative zero of $p(x)=(x-7)(x+10)$.

-10

The first factor vanishes at 7. The second vanishes at -10, which is the negative zero.

Example 2

Find the degree of $2x^4-x^2+8$.

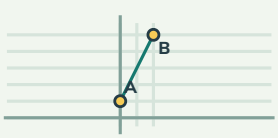
4

The coefficient of x^4 is nonzero.

Can two different polynomials share their zeros?

Take $p(x) = (x - 2)(x + 3)$ and $q(x) = 4p(x)$. Each product is zero at 2 and -3. The factor 4 changes neither root.

At $x = 0$ their values are -6 and -24. Matching zeros alone do not determine the leading coefficient.



A (0, 1)
B (2, 5)

Find rise and run between the points.

Rise 4, run 2. Slope = 4/2 = 2.

ILLUSTRATED PROBLEM SHEET

Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
$x = \pm 3$.

$x^2 + 9 = 0$
no real roots.

CHAPTER PROBLEM

Find the negative zero of $p(x)=(x-4)(x+7)$.

-7. For a product to vanish, one factor must vanish. The two candidates are 4 and −7. Only −7 is negative.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can a nonzero quadratic have three distinct real zeros?	No. Three distinct linear factors would require degree at least three.
02 What happens to $x^2 - 6x + k = 0$ at $k = 9$?	It becomes $(x - 3)^2 = 0$. The two root values coincide at 3.
03 Which is a polynomial in x? A) x^2+1/x B) $\sqrt{x}+2$ C) $3x^3-2x+7$ D) $x^{-2}+4$	$3x^3-2x+7$ Only the third has nonnegative integer exponents.
04 Find the degree of $7x^4-x^2+8$.	4 The coefficient of x^4 is nonzero.
05 For $p(x)=2x+16$, find its zero.	-8 At a zero the output is zero. Solve $2x + 16 = 0$ to get $x = -8$. Substituting gives $-16 + 16 = 0$.
06 A parabola touches the x-axis once. How many distinct real zeros does it have? A) 0 B) 1 C) 2 D) Infinitely many	1 The single touching point supplies one distinct x-coordinate.

$$y = 3x - 2$$
$$\downarrow$$
$$10 = 3x - 2$$
$$\downarrow$$
$$x = 4$$

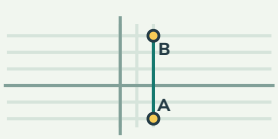
Which input produces output ten?

Undo subtraction, then divide by three. Check: $3 \times 4 - 2 = 10$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain why x^2+3 has no real zeros.	A real square is nonnegative. Adding a positive constant leaves a strictly positive result.
08 Distinguish a zero of p from the value p(0).	A zero is an input whose output is zero. p(0) is the output at input zero and need not itself be zero.
09 Find p(4) when $p(x)=x^2-16$.	0 Substitution gives $4^2 - 16 = 0$, making 4 a zero of the polynomial.
10 Find the negative zero of $p(x)=(x-5)(x+8)$.	-8 The second factor vanishes at the negative value.
11 Find p(6) when $p(x)=x^2-36$.	0 Substitute 6 for both occurrences of x in $x \times x$. Then $p(6) = 6 \times 6 - 36 = 0$, so 6 is a zero of p.



A (2, -2)
B (2, 3)

Does this vertical line have slope zero?

No. Its run is zero, so rise divided by run is undefined. A horizontal line has slope zero.

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