MATHS FIELD GUIDE · GRADE 10

Nonmonic quadratic factorisation

OPENING PROBLEM

Which products make the middle term?

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

02 Choose a quadratic method
03 Rules and worked cases
04 From the givens to the answer
05 Recover coefficients from two zeros
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
$x = \pm 3$.

$x^2 + 9 = 0$
no real roots.

CHAPTER PROBLEM

For $(2x-1)(x-4)=0$, find the integer root.

4. Set the second factor to zero.

MATHS REFERENCE · GRADE 10

Nonmonic quadratic factorisation

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Split the middle	For $3x^2 + 10x + 8$, choose 6 and 4: sum 10, product 24.
Group	$3x^2 + 6x + 4x + 8 = 3x(x + 2) + 4(x + 2) = (3x + 4)(x + 2)$.
Solve and check	Zeros $-4/3$ and -2 have sum $-10/3$ and product $8/3$, matching the coefficients.

Read the representations

The leading coefficient affects both factor pairs

For ax^2+bx+c, $a \neq 0$, factor out any common factor first. To split the middle term, find p,q with $pq=ac$ and $p+q=b$.

Then group the four terms into a common binomial factor. Verify the final product by expansion before solving.

Split the middle term

Stage	Expression
Original	$2x^2+7x+3$
Split 7 as 6+1	$2x^2+6x+x+3$
Group	$2x(x+3)+1(x+3)$
Factor	$(2x+1)(x+3)$

A fractional root is a valid root

A factor $2x+1=0$ gives $x=-1/2$. Roots need not be integers even when coefficients are integers.

Keep the zero-product condition and check roots in the original equation.

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and -3. Each makes a different factor vanish.

WORKED EXAMPLES

From the givens to the answer

Solve $3x^2-10x+3=0$

1. The product ac is 9 and the middle sum is −10, giving −9 and −1.
2. Group $3x(x-3)-1(x-3)=(3x-1)(x-3)$.
3. Roots are 1/3 and 3.

Remove a common factor first

1. For $4x^2+10x+4=0$, divide both sides by 2.
2. Factor $2x^2+5x+2=(2x+1)(x+2)$.
3. Roots are −1/2 and −2.

Do the middle-split numbers always multiply to c?

1. When the leading coefficient is not one, they multiply to ac.
2. Their sum still equals b.

Check another case

Example 1

For $(3x-1)(x-8)=0$, find the non-integer root.

1/3

Solve $3x-1=0$.

Example 2

Which expands to $2x^2+19x+9$?

$(2x+1)(x+9)$ · $(2x-1)(x+9)$ · $(x+1)(x+9)$ · $(2x+9)(x+9)$

$(2x+1)(x+9)$

Expand to check all three coefficients.

Find two middle terms that work

For $6x^2 + x - 2$, the product ac is −12. Use 4 and −3, which add to 1. Group as $2x(3x + 2) - 1(3x + 2)$.

The factors are $(2x - 1)(3x + 2)$. They give roots 1/2 and −2/3.

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

ILLUSTRATED PROBLEM SHEET

Recover coefficients from two zeros

For $ax^2 + bx + c$ with $a \neq 0$, the two zeros (counting repeats) sum to $-b/a$ and multiply to c/a.

1

Root 1	Root 2	Sum	Product
2	5	7	10
−2	5	3	−10

Write the monic polynomial for the second row.

$x^2 - 3x - 10$. The sign of the middle coefficient is the opposite of the sum.

2

Zeros −1 and 4
↓
$(x + 1)(x - 4)$
↓
$x^2 - 3x - 4$

What if the leading coefficient must be 3?

Triple the whole polynomial to obtain $3x^2 - 9x - 12$. Its zeros stay fixed.

3

$2x^2 - 5x + c$
↓
One zero = 2
↓
$8 - 10 + c = 0$

Find c and the second zero.

$c = 2$. The sum is 5/2, so the other zero is 1/2. Their product is 1.

4

Sum = 0
↓
Product = −9
↓
$x^2 - 9$

Can opposite zeros make the linear coefficient vanish?

Yes. The zeros 3 and −3 add to zero.

Roots 0 and 7
monic polynomial $x^2 - 7x$.

Roots −7 and 7
monic polynomial $x^2 - 49$.

CHAPTER PROBLEM

For $(3x-1)(x-5)=0$, find the non-integer root.

1/3. Solve $3x-1=0$.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 What happens to $x^2 - 6x + k = 0$ at $k = 9$?	It becomes $(x - 3)^2 = 0$. The two root values coincide at 3.
02 A monic quadratic has zeros that add to 6 and multiply to 9. How many distinct zeros?	$x^2 - 6x + 9 = (x - 3)^2$, so one distinct zero.
03 Which expands to $2x^2+15x+7$? A) $(2x+1)(x+7)$ B) $(2x-1)(x+7)$ C) $(x+1)(x+7)$ D) $(2x+7)(x+7)$	$(2x+1)(x+7)$ Expand to check all three coefficients.
04 For $2x^2+17x+8$, what product is required when splitting the middle term? A) 16 B) 8 C) 17 D) 64	16 The split product is ac.
05 For $(2x+18)(x-1)=0$, find its negative root.	-9 Set $2x + 18 = 0$ to obtain -9. The other factor gives $+1$, so it does not answer the request for a negative root.
06 A factorisation is $(2x+1)(x+3)$. Find its x^2 coefficient.	2 Multiply the leading terms.

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain how to verify $(2x+1)(x+4)$ before solving.	Expand leading, middle and constant terms. Compare with $2x^2+9x+4$. Use the zero-product rule only if the equation has zero on its other side.
08 A learner rejects $x=1/5$ because the original quadratic has integer coefficients. Explain the mistake.	Integer coefficients do not force integer roots. A linear factor $5x-1$ gives the fractional root $1/5$. Substitute the fraction to verify it.
09 For $(2x-1)(x-5)=0$, find the integer root.	5 The first factor gives $x = 1/2$ and the second gives $x = 5$. Both solve the equation, but only 5 is an integer.
10 For $(3x-1)(x-6)=0$, find the non-integer root.	1/3 One factor must be zero. Solving $3x - 1 = 0$ gives $x = 1/3$. The other root, 6, is an integer.
11 For $(2x-1)(x-7)=0$, find the integer root.	7 Set the second factor to zero.

Sum = −1
↓
Product = −6
↓
$x^2 + x - 6$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are $(x - 2)$ and $(x + 3)$.

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