MATHS FIELD GUIDE · GRADE 10

Modelling a pair of equations

OPENING PROBLEM

Which x satisfies both conditions?

$$y = x + 3$$
$$\downarrow$$
$$x + y = 11$$
$$\downarrow$$
$$x + (x + 3) = 11$$

02 Substitute the whole expression
03 Rules and worked cases
04 From the givens to the answer
05 Make one coefficient cancel
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Substitute the whole expression

An expression may replace a variable only when the two are equal. Brackets keep all its terms together.

1

$$y = x + 3$$
$$\downarrow$$
$$x + y = 11$$
$$\downarrow$$
$$x + (x + 3) = 11$$

Which x satisfies both conditions?

2x + 3 = 11 gives x = 4. Then y = 7. Both sum to eleven.

2

$$x = 2y - 1$$
$$\downarrow$$
$$3x + y = 18$$
$$\downarrow$$
$$3(2y - 1) + y = 18$$

Why multiply both terms in the bracket?

The 3 multiplies all of x. This gives 7y – 3 = 18, so y = 3 and x = 5.

3

$$y = 4 - x$$
$$\downarrow$$
$$2x + 2y = 8$$
$$\downarrow$$
$$8 = 8$$

Why does a true constant remain?

The second equation doubles the first relationship. Every point on y = 4 – x works.

4

$$y = 4 - x$$
$$\downarrow$$
$$x + y = 7$$
$$\downarrow$$
$$4 = 7$$

Why is there no solution here?

The first condition fixes the sum at 4. The second asks for a different sum.

y = x, x + y = 8
(4,4).

y = −x, x + y = 8
impossible.

CHAPTER PROBLEM

A notebook and a pen cost ₹46. Two notebooks and one pen cost ₹74. Find the notebook price in rupees.

28. Subtract the first purchase from the second to get n = 28.

MATHS REFERENCE · GRADE 10

Modelling a pair of equations

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Count and cost	Five tickets cost ₹34 at ₹8 adult and ₹5 child. Let a + c = 5 and 8a + 5c = 34.
Use the all-cheap baseline	Five child tickets cost ₹25. The extra ₹9 buys three ₹3 upgrades: three adults and two children.
Feasibility check	₹35 gives ten extra rupees, which cannot be split into whole ₹3 upgrades. No whole-ticket mix works.

Read the representations

Define the two unknowns with units

Let n be the price of one notebook in rupees and p the price of one pen in rupees.

Each purchase gives one total-cost relationship. The same item must keep the same price in both purchases.

Use the item counts as coefficients, and the total cost as the right side.

Purchase 1

Notebook
₹nNotebook
₹nPen
₹p
Total: ₹85

Purchase 2

Notebook
₹nPen
₹pPen
₹pPen
₹p
Total: ₹80

The two receipt equations are 2n + p = 85 and n + 3p = 80.

Two facts must provide different information

If the second purchase is exactly twice the first, its total only repeats the same condition.

To determine two unknown prices uniquely, the conditions must be independent.

After solving, check that both original purchases have the stated total.

A feasible answer depends on what is being measured

Counts such as tickets must be non-negative integers. Prices or measured lengths can have fractional values when the context permits.

A mathematically solved pair may violate those restrictions. Do not round it into a different pair.

Check units and whether every result is possible in the original situation.

Time (h)	Distance (km)
1	18
2	36
2.5	45

At steady speed, how far in 2½ hours?

Two hours give 36 km and half an hour adds 9 km. Total 45 km.

WORKED EXAMPLES

From the givens to the answer

Can a total count be used as a price equation?

A learner writes a + c = 1400 when a and c represent adult and child ticket counts, but 1400 is revenue in rupees.

1. The sum a + c counts tickets, so its right side must be a ticket count.
2. A revenue equation multiplies each count by its ticket price.
3. Keep the units consistent on both sides.

Model 14 tickets with prices ₹120 and ₹70, total ₹1430

1. Let a and c be adult and child ticket counts: a + c = 14 and 120a + 70c = 1430.
2. Substitute c = 14 − a: 120a + 980 − 70a = 1430.
3. Then 50a = 450, so a = 9 and c = 5.
4. Both counts are non-negative integers; 9 × 120 + 5 × 70 = 1430.

Check a claim about 10 tickets and ₹550 revenue

1. Suppose adult tickets cost ₹100 and child tickets ₹70. Even ten child tickets cost ₹700.
2. Revenue ₹550 is below that minimum, so the data are infeasible.
3. The equations would give a = −5 and c = 15. Negative adult tickets confirm the contradiction; do not round or accept the counts.

Check another case

Example 1

Which is required for a valid contextual answer?

Both equations only, regardless of units · Units, both conditions and feasibility · Rounding every value · Making both unknowns positive by changing signs

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Units, both conditions and feasibility

The pair must satisfy the original conditions and make sense for the quantities.

Example 2

Describe what to write before constructing a pair of equations from a story.

Define each unknown precisely.

Include the unit or whether it is a count.

Translate each distinct condition using those same definitions.

600 m in 5 min
↓
120 m per min
↓
7.2 km per hour

Why divide before multiplying?

Find distance in one minute, then scale to sixty minutes and convert metres to kilometres.

ILLUSTRATED PROBLEM SHEET

Make one coefficient cancel

Multiplying an entire equation by a nonzero number preserves its line.

1

$$2x + y = 11$$
$$\downarrow$$
$$x - y = 1$$
$$\downarrow$$
$$3x = 12$$

Which variable cancels when you add?

The y terms. x = 4 and y = 3. Check the difference is one.

2

$$3x + 2y = 16$$
$$\downarrow$$
$$x + 2y = 8$$
$$\downarrow$$
$$2x = 8$$

Why subtract the second row?

The y coefficients already match. Subtracting gives x = 4, then y = 2.

3

$$x + 2y = 7$$
$$\downarrow$$
$$3x + y = 11$$
$$\downarrow$$
$$3x + 6y = 21$$

How do you prepare matching coefficients?

Triple the first equation. Subtract the second to get 5y = 10, hence y = 2 and x = 3.

4

First	Second	Second − twice first
x + y = 4	2x + 2y = 8	0 = 0
x + y = 4	2x + 2y = 9	0 = 1

What does eliminating both variables tell you?

The first pair describes a single line. The second pair has no shared point.

Equal coefficients with the same sign subtract the equations.

Equal coefficients with opposite signs add the equations.

CHAPTER PROBLEM

Two pens and one pencil cost ₹26. One pen and one pencil cost ₹16. Find one pen's price.

10. Subtract the second purchase from the first.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A pen costs ₹2 more than a pencil. Two pencils and a pen cost ₹17. Find each price.	Pencil x, pen x + 2. Then 3x + 2 = 17, so the prices are ₹5 and ₹7.
02 For adult count a and child count c, which correctly states 18 tickets in total? A) a + c = 18 B) 18a + c = 0 C) a − c = 18 D) a + c = revenue	a + c = 18 Both variables count tickets, so their sum is the total count.
03 Could you multiply just the x term by three to prepare the first equation?	No. Every term on both sides must scale together.
04 If x and y are ticket counts, which solved pair is feasible? A) (3.5,6.5) B) (−2,12) C) (4,6) D) (10,−1)	(4,6) Ticket counts must be non-negative integers.
05 A sum is 22 and the difference, larger minus smaller, is 8. Find the larger number.	15 Add the equations to get twice the larger number = 30.
06 A rectangle has perimeter 34 cm and length 3 cm more than width. Find width in cm.	7 L + W = 17 and L = W + 3 give 2W = 14.

Pack	Price
250 g	₹40
400 g	₹60

Which pack costs less per 100 g?

₹16 versus ₹15. The 400 g pack costs less per 100 g.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 A second equation is exactly three times the first, including its total. It gives... A) Independent new information B) A unique price automatically C) The same condition again D) An impossible equation automatically	The same condition again The second equation describes the same relationship.
08 There are 11 tickets: adults at ₹80, children at ₹50. Revenue is ₹730. Find the adult count.	6 80a + 50(11 − a) = 730 gives 30a = 180, so a = 6.
09 Why should a fractional solution for a ticket count not be rounded and reported as the answer?	Tickets are counted as whole items. Rounding changes the values and generally breaks one or both equations. A non-integer pair means the stated data or model needs review.
10 Explain how to verify prices found from two purchase totals.	Check that the unknowns still mean the same unit prices. Recalculate each purchase using its own item counts. Confirm both totals and that the prices are allowed by the situation.
11 A sum is 17 and the larger exceeds the smaller by 5. Find the smaller number.	6 The equations give larger 11 and smaller 6.

Time (min)	Water (L)
0	3
1	5
2	7

Is water volume proportional to time?

It grows by 2 L each minute but starts at 3 L. Doubling time need not double the amount.

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