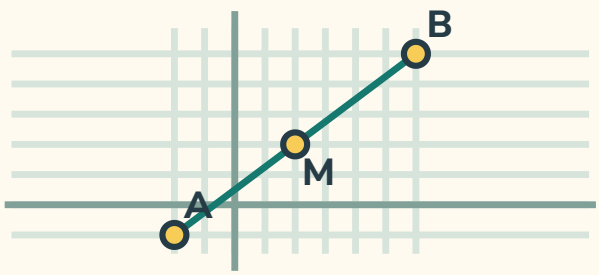
MATHS FIELD GUIDE · GRADE 10

Internal division and midpoint

OPENING PROBLEM

A(−2, −1) and B(6, 5) share which midpoint?



A (-2, -1)
M (2, 2)
B (6, 5)

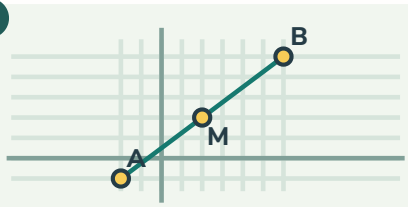
02 Locate a point along a segment
03 Rules and worked cases
04 From the givens to the answer
05 A parallel cut creates two similar triangles
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Locate a point along a segment

Moving a fraction of the full displacement locates the corresponding internal point.

1

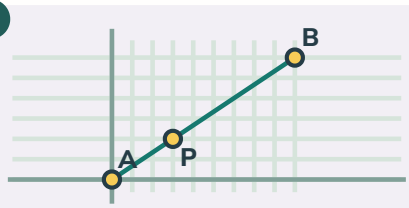


A (-2, -1)
M (2, 2)
B (6, 5)

A(−2, −1) and B(6, 5) share which midpoint?

Half the displacement (8, 6) is (4, 3). Add it to A to get M(2, 2).

2



A (0, 0)
P (3, 2)
B (9, 6)

For A(0, 0) and B(9, 6), where is the 1:2 division point from A?

Travel one of three equal steps, so P = (3, 2).

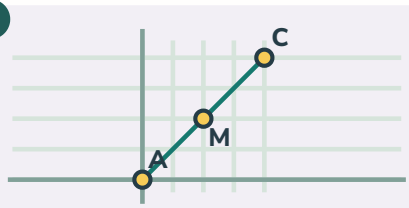
3

A = (−3, 1)
↓
M = (1, 4)
↓
B = (5, 7)

Recover the far endpoint from a midpoint.

From A to M add (4, 3). Add the same vector again to reach B.

4



A (0, 0)
M (2, 2)
C (4, 4)

How can midpoints test a quadrilateral?

Compare both diagonal midpoints. If both bisect each other in a simple quadrilateral, it is a parallelogram.

Midpoint of (0,0) and (8,4)
(4,2).

One quarter of the same trip
(2,1).

CHAPTER PROBLEM

A=(3,2), B=(15,8). P divides AP:PB=1:2. Find x(P).

7. P lies one third of the way from A to B. Its x-coordinate is 3 + (15 − 3)/3 = 7.

MATHS REFERENCE · GRADE 10

Internal division and midpoint

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Internal ratio	From A(0,1) to B(9,7), the change is (9,6). AP:PB = 1:2 puts P one third along at (3,3).
Reverse the ratio	AP:PB = 2:1 puts P two thirds along at (6,5), closer to B.
Midpoint special case	Equal parts give (4.5,4), the coordinate averages. Each coordinate lies between the endpoints' values.

Read the representations

An internal ratio divides the displacement

For A(x_1,y_1), B(x_2,y_2), let AP:PB=m:n with m,n positive. P lies fraction m/(m+n) of the way from A to B.

Thus P=A+[m/(m+n)](B−A), giving $((nx_1+mx_2)/(m+n),(ny_1+my_2)/(m+n))$. The opposite endpoint gets the weight m.



AP:PB = 1:2; P is 1/3 of the way from A.

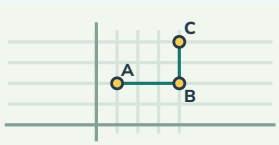
Midpoint is the equal-ratio special case

For m=n, average corresponding coordinates.

Check that an internal point lies between the endpoint coordinates; do not mix AP:PB with PB:AP.

Along A(1,2) to B(7,8)

AP:PB	Fraction from A	Point
1:1	1/2	(4,5)
1:2	1/3	(3,4)
2:1	2/3	(5,6)



A (1, 2)
B (4, 2)
C (4, 4)

Describe A to C via B.

Go three right, then two up. The total route length is five grid units.

WORKED EXAMPLES

From the givens to the answer

Divide A(−2,3), B(8,−2) in ratio AP:PB=2:3

1. The fraction from A is 2/5.
2. x=−2+(2/5)(10)=2; y=3+(2/5)(−5)=1.
3. P=(2,1). The displacement AP=(4,−2) is 2/5 of AB=(10,−5).

Whose share is closer to A?

A is (−2, 1) and B is (10, 7). Point P divides AB with AP:PB = 1:2. Locate P.

1. The displacement is (12, 6). Travel one of three equal parts from A, adding (4, 2) to reach (2, 3). Two more equal steps reach B.

Check another case

Example 1

A=(1,7), B=(7,16). AP:PB=2:1. Find y(P).

13

P is two thirds of the way from A.

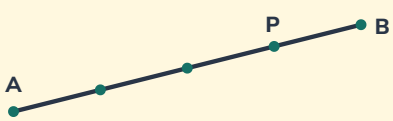
Example 2

Find midpoint x-coordinate for A=(−8,4), B=(14,8).

3

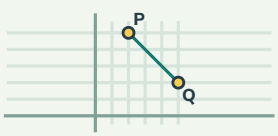
Average the x-coordinates: (−8 + 14)/2 = 3. Both endpoints are eleven horizontal units from this value.

Reverse a division ratio



Split the segment from (0, 0) to (12, 8) in the ratio 3:1 from the origin. Three quarters of the displacement lands at (9, 6).

Swapping the ratio to 1:3 lands at (3, 2). These two points have midpoint (6, 4), also the midpoint of the original segment.



P (2, 5)
Q (5, 2)

Are the points the same if the coordinates swap?

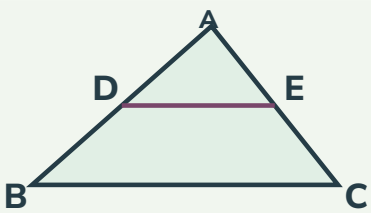
No. Each has its horizontal and vertical distances exchanged.

ILLUSTRATED PROBLEM SHEET

A parallel cut creates two similar triangles

D lies on AB, E lies on AC and DE is parallel to BC unless the question asks you to test it.

1



DE ∥ BC

Use AB = 8, AC = 6. AD = 4. Find AE.

D is halfway to B. The smaller triangle is at half scale, so AE = 3.

2

AD = 3, DB = 6
↓
AE = 4
↓
AD/DB = AE/EC

Recover the missing segment EC.

The lower piece is twice the upper piece on AB. EC must be twice 4, which is 8.

3

Side	Upper	Lower
AB	3	5
AC	6	10

If parallelism was not given, can you deduce it?

Yes. 3/5 = 6/10. The converse applies because D and E are internal points.

4

AD/AB = 2/5
↓
Area ADE / Area ABC = 4/25

What fraction of the large area remains below DE?

1 − 4/25 = 21/25. The area factor is the square of the length factor.

AD:DB = 1:3
AD:AB = 1:4.

AD:AB = 3:4
AD:DB = 3:1.

CHAPTER PROBLEM

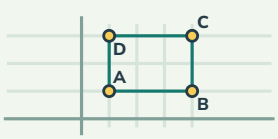
A=(1,4), B=(7,13). AP:PB=2:1. Find y(P).

10. P is two thirds of the way from A.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Does swapping AP:PB from 1:2 to 2:1 keep P fixed?	Only an equal division would stay fixed on reversal. On the segment from (0,0) to (9,6) the new point is (6,4).
02 May you set AD/DB equal to AE/AC for a parallel cut?	Those fractions compare different parts. Match part to remainder on both sides, or part to whole on both.
03 Find midpoint x-coordinate for A=(−6,4), B=(12,8). 	3 The x-coordinates are 18 apart. Move halfway, or nine units right of −6, to reach 3.
04 A=(−4,2),B=(8,7). P is on the y-axis. Find AP/PB.	0.5 x=0 is one third from −4 to 8, so AP:PB=1:2.
05 In AP:PB=m:n, which weight multiplies x_1 in the section numerator? A) m B) n C) m+n D) m−n	n P is a weighted average: nx_1+mx_2 over m+n.
06 Derive the midpoint formula from internal section.	Set m=n in both coordinate formulae. Cancel the common weight to obtain coordinate averages.



A (1, 1)
B (4, 1)
C (4, 3)
D (1, 3)

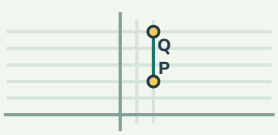
How can you recognise this rectangle?

Opposite sides have the same lengths. Horizontal and vertical neighbours meet at right angles.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Why must the two ratio parts be positive for internal division?	A positive fraction m/(m+n) then lies strictly between 0 and 1. This places P between A and B rather than on an extension.
08 A=(4,2), B=(16,8). P divides AP:PB=1:2. Find x(P).	8 P is one third of the way from A to B.
09 A=(1,5), B=(7,14). AP:PB=2:1. Find y(P).	11 P is two thirds of the way from A.
10 A=(0,0), B=(6,12). AP:PB=1:2. Find y(P).	4 Take one third of the displacement.
11 A=(6,2), B=(18,8). P divides AP:PB=1:2. Find x(P).	10 P is one third of the way from A to B.



P (2, 2)
Q (2, 5)

Must both coordinates change when a point moves?

No. A vertical move keeps x fixed and adds three to y.

BANANALEARN LEARNING LIBRARY

Where will your maths take you next?

GRADE 10 MATHS

Choose your next problem.

Explore more illustrated guides and practise a related topic.

Browse Grade 10 guides →

Keep exploring

GRADE 10 · TOPIC GUIDE

Distance in the coordinate plane

Open guide →

GRADE 10 · TOPIC GUIDE

Coordinate geometry applications

Open guide →

Visit bananalearn.com →