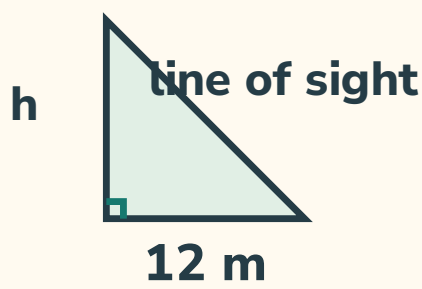
MATHS FIELD GUIDE · GRADE 10

Heights and distances with two triangles

OPENING PROBLEM

At 45° elevation over 12 m of level ground, how far above eye level is the target?



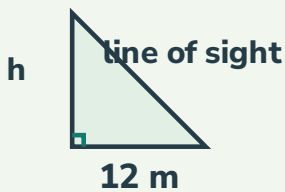
02 Turn a sight line into a right triangle
03 Rules and worked cases
04 From the givens to the answer
05 Rebuild the exact trigonometric values
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Turn a sight line into a right triangle

Measure elevation from a horizontal line at eye level. Add eye height only after finding the vertical rise.

1



At 45° elevation over 12 m of level ground, how far above eye level is the target?

tan 45° = h/12 = 1, so h = 12 m.

2

Horizontal distance = 10 m
↓
Elevation = 60°
↓
Rise = 10√3 m

The observer's eyes are 1.5 m above ground. What is the full height?

10√3 + 1.5 metres. Adding eye height changes the reference level.

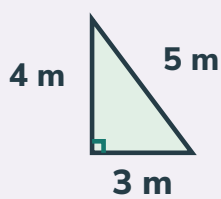
3

Near distance x and far x + 20
↓
h = x√3
↓
h = (x + 20)/√3

The near and far angles are 60° and 30°. Find x.

Equate the two heights. 3x = x + 20, so x = 10 m and h = 10√3 m.

4



A 5 m ladder reaches 4 m up a wall. What is the horizontal distance?

√(25 − 16) = 3 m. The sight-line or ladder length is distinct from the ground distance.

Twice the distance at the same elevation angle
twice the rise.

Same distance and larger acute angle
greater rise.

CHAPTER PROBLEM

At two points on the same side of a tower, elevations are 60° near and 30° far. Their separation is 8 m. Find the near ground distance.

4. Let x metres be the near distance. The height is x√3 from the near point and (x + 8)/√3 from the far point. Equating them gives 3x = x + 8, so x = 4 m.

MATHS REFERENCE · GRADE 10

Heights and distances with two triangles

Rules, conditions and worked cases

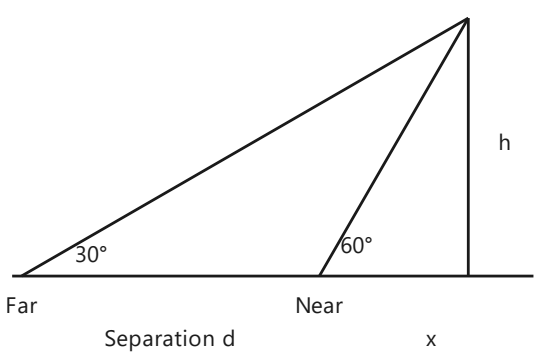
Recognise the case	Mathematical reasoning
Same side of tower	Near distance x and separation d give $h = x \tan\alpha = (x + d) \tan\beta$, with α the larger angle.
Opposite sides	If points enclose the tower and are d apart, horizontal distances are x and $d - x$. Use $h = x \tan\alpha = (d - x) \tan\beta$.
Equal eye heights	Solve for the vertical rise above eye level, then add the common eye height once.

Read the representations

Two observations share one unknown height

Draw the horizontal ground and the two lines of sight. Mark which observation is nearer and which distance is given.

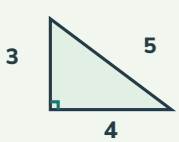
For two ground points on the same side of a vertical tower, let the near distance be x and their separation d. Then $h = x \tan\theta_{near} = (x+d)\tan\theta_{far}$.



Eye height shifts the vertical difference

If the observer's eyes are e above the base level, the right triangles use H−e, not the full tower height H.

For depression angles, use the horizontal through the observer; parallel horizontals give equal elevation and depression angles. Problems here use at most two right triangles and standard 30°,45°,60° angles.



For the acute angle opposite 3, find sine and cosine.

$\sin\theta = 3/5$ and $\cos\theta = 4/5$. The hypotenuse is the denominator in each.

WORKED EXAMPLES

From the givens to the answer

Near elevation 60°, far elevation 30°, separation 20 m

1. Let the near ground distance be x and tower height h. Then $h=x\sqrt{3}=(x+20)/\sqrt{3}$.
2. Multiplying by $\sqrt{3}$ gives $3x=x+20$, so $x=10$.
3. $h=10\sqrt{3}$ m. The far distance is 30 m; $\tan 30°=h/30$ confirms the result.

Compare two viewing positions

From level ground, a tower top is seen at 30°. Walk 20 m straight towards its base and the angle becomes 60°. Find its height above eye level.

1. Let x be the nearer horizontal distance. Then $h = x\sqrt{3} = (x + 20)/\sqrt{3}$. Thus $3x = x + 20$, giving $x = 10$ m and $h = 10\sqrt{3}$ m.

Check another case

Example 1

At two points on the same side of a tower, elevations are 60° near and 30° far. Their separation is 14 m. Find the near ground distance.

7

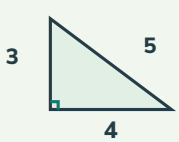
Let x be the near distance. The same height equals $x\sqrt{3}$ and $(x + 14)/\sqrt{3}$. Hence $3x = x + 14$ and $x = 7$ m.

Example 2

Two ground points on the same side of a vertical tower have elevations 60° near and 30° far and separation 16 m. Write tower height as $c\sqrt{3}$ m. Find c.

8

With x metres to the near point, the heights are $x\sqrt{3}$ and $(x + 16)/\sqrt{3}$. Thus $3x = x + 16$ and $x = 8$. The height is $8\sqrt{3}$ m.



Use the angle opposite 4 instead. What happens to tangent?

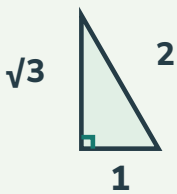
It becomes 4/3. For the other angle it was 3/4.

ILLUSTRATED PROBLEM SHEET

Rebuild the exact trigonometric values

Use half an equilateral triangle or half a square. At 0° and 90° use unit-circle coordinates.

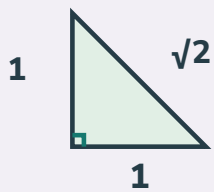
1



Which acute angle faces the short leg?

30°. Thus sin 30° = 1/2 and cos 30° = √3/2.

2



Why is tan 45° exactly one?

The legs match. Sine and cosine each equal 1/√2.

3

θ	0°	90°
cos θ	1	0
sin θ	0	1

At which endpoint does tan θ fail?

At 90°. Cosine is zero, so the quotient sine/cosine is undefined.

4

$$\theta = 60°$$
$$\downarrow$$
$$\sin^2\theta = 3/4$$
$$\downarrow$$
$$\cos^2\theta = 1/4$$

Check the identity at an exact angle.

Three quarters plus one quarter equals one. The individual ratios are square roots of these values.

sin 30° equals cos 60°
both 1/2.

sin 60° equals cos 30°
both √3/2.

CHAPTER PROBLEM

Two ground points on the same side of a vertical tower have elevations 60° near and 30° far and separation 10 m. Write tower height as c√3 m. Find c.

5. Let x metres be the near distance. The same height is x√3 and (x + 10)/√3. Equating gives 3x = x + 10, so x = 5. The height is 5√3 m.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can the two-station equation use x + 20 if the tower lies between the stations?	No. On opposite sides the two ground distances add to the station separation.
02 Is cosec 0° equal to zero?	No. It is 1/sin 0°, so the denominator is zero and it is undefined.
03 Two ground points lie on opposite sides of a tower and are 32 m apart. Both elevations are 45°. Find the tower height.	16 Both distances equal h, so total separation is 2h.
04 Two observers stand on the same side of a tower at positive distances x and x+d, with d>0. Which is nearer? A) x+d B) x C) Both D) Not enough information even if d>0	x For d>0, x is smaller.
05 An angle of depression is 30°. Find the corresponding angle of elevation in degrees.	30° The two horizontal lines are parallel, giving alternate angles.

$\sin\theta = 5/13$
↓
$\text{cosec}\,\theta = 13/5$

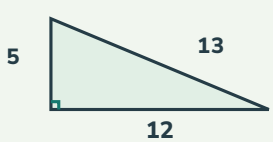
Why does the fraction invert?

Cosecant is hypotenuse over opposite. Its product with sine is one wherever both are defined.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
06 Derive the height for elevations 60° and 30° at same-side points d apart.	Let near distance x; set $h=x\sqrt{3}=(x+d)/\sqrt{3}$. Obtain $x=d/2$ and $h=d\sqrt{3}/2$. State the equal ground level and vertical tower assumptions.
07 Why does using H rather than H−e give a wrong model when eye height e is nonzero?	The line of sight begins at eye level. The opposite leg is the vertical difference between the top and the eye, so add e only after solving.
08 At two points on the same side of a tower, elevations are 60° near and 30° far. Their separation is 10 m. Find the near ground distance.	5 $x\sqrt{3}=(x+d)/\sqrt{3}$ gives $2x=d$.
09 Two ground points on the same side of a vertical tower have elevations 60° near and 30° far and separation 12 m. Write tower height as $c\sqrt{3}$ m. Find c.	6 Let x be the near distance. Then $x\sqrt{3} = (x + 12)/\sqrt{3}$, so $3x = x + 12$ and $x = 6$. The height is $6\sqrt{3}$ m.



Use the triangle to check $\sin^2\theta + \cos^2\theta$.

The numerator is 25 + 144 = 169 over 169. The sum is one.

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