MATHS FIELD GUIDE · GRADE 10

HCF and LCM in context

OPENING PROBLEM

Check the HCF and LCM with prime factors.

Factor	84	30
2	2 copies	1 copy
3	1 copy	1 copy
Other	7	5

02 Divisibility without listing every multiple
03 Rules and worked cases
04 From the givens to the answer
05 Use the prime powers as building blocks
06 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Divisibility without listing every multiple

A remainder, a common divisor and a repeated interval answer different questions.

1

$$84 = 2 \times 30 + 24$$
$$\downarrow$$
$$30 = 1 \times 24 + 6$$
$$\downarrow$$
$$24 = 4 \times 6$$

Find HCF(84, 30) by remainders.

The last nonzero remainder is 6. Each division preserves the common divisors.

2

Factor	84	30
2	2 copies	1 copy
3	1 copy	1 copy
Other	7	5

Check the HCF and LCM with prime factors.

HCF = 2 × 3 = 6. LCM = 4 × 3 × 5 × 7 = 420.

3

Two bells: 12 and 18 min
↓
LCM = 36 min

If both ring at 9:00, when next together?

9:36. A shared future time needs a multiple of both intervals.

4

30 red and 42 blue
↓
Identical packs
↓
HCF = 6 packs

How many of each colour in each largest-count pack arrangement?

Five red and seven blue. Every pack has the same contents with none left over.

HCF(18,24)
6.

LCM(18,24)
72; their product is 18 × 24.

CHAPTER PROBLEM

There are 36 red and 60 blue beads. What is the greatest number of identical packs using all beads?

12. HCF(36,60) = 12 packs.

MATHS REFERENCE · GRADE 10

HCF and LCM in context

Rules, conditions and worked cases

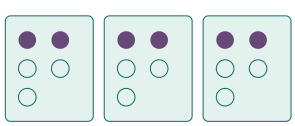
Recognise the case	Mathematical reasoning
Greatest equal kit count	42 red and 56 blue pieces form 14 identical kits. Each has three red and four blue.
First repeated meeting	Two signals every 12 and 18 seconds meet again after LCM = 36 seconds.
Same remainder	A divisor leaves 2 in 38 and 50. It divides 36 and 48. Greatest candidate 12 works since 2 < 12.

Read the representations

Grouping asks what divides every supply

If identical packs use every item, the number of packs must divide each supply.

The greatest possible pack count is the HCF. The pack contents are found by dividing each supply by that count.



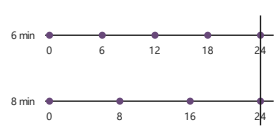
Each pack 2 purple + 3 mint

3 packs use all 6 purple and all 9 mint.

Repeating events ask for a shared multiple

A repeat every 6 minutes occurs at 0, 6, 12, 18... A repeat every 8 occurs at 0, 8, 16, 24...

The first shared positive time is 24, the LCM. Time 0 is the starting coincidence, not the next one.



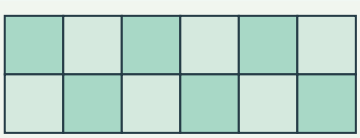
Next coincidence after time 0: 24 minutes

Remainders change the numbers to compare

If N leaves remainder 2 when divided by 5 and 7, then N − 2 is divisible by both.

N = 35k + 2 for an integer k, with N in the stated range. A remainder condition alone may not determine one answer.

1. **Remove the common remainder**N − 2 is a multiple of both 5 and 7.
2. **Find the common cycle**LCM(5,7) = 35.
3. **Restore the remainder**Possible values include 2, 37, 72... Choose using the range.



What factor pair builds twelve?

Two rows of six. Turning the rectangle gives the same pair reversed.

WORKED EXAMPLES

From the givens to the answer

Make the largest number of identical packs from 42 pencils and 56 erasers

1. HCF(42,56) = 14, so make 14 packs.
2. Each pack contains 42/14 = 3 pencils and 56/14 = 4 erasers.
3. 14×3 = 42 and 14×4 = 56: every item is used.

Check another case

Example 1

A divisor leaves remainder 5 on dividing both 65 and 95. Find its greatest possible value.

30

It divides 60 and 90; HCF = 30, and 5 < 30.

Example 2

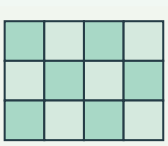
Find all integers from 40 to 90 that leave remainder 1 on division by 6 and 8.

N − 1 is a multiple of LCM(6,8) = 24.

The qualifying values are 49 and 73; check both conditions.

Work backwards and test the limits

Problem	Derivation
A floor is 168 cm by 252 cm. What is the biggest square tile that covers it without cutting?	252 − 168 = 84 and 168 = 2 × 84. The HCF is 84 cm. Two rows by three columns use six square tiles.
Two timers tick every 8 and 14 seconds. The first ticks now; the second next ticks in 3 seconds. Will they tick together?	The first ticks at multiples of 8, all even. The second ticks at 3 + 14k, always odd. Their starting offset prevents a meeting.
Can two positive integers have HCF 12 and LCM 50?	No. The HCF divides each integer and therefore divides their LCM. Since 12 does not divide 50, the proposed pair is impossible.



Which other pair also uses twelve?

Three and four. The long skinny arrangement 1 by 12 completes the list.

ILLUSTRATED PROBLEM SHEET

Use the prime powers as building blocks

Prime exponents record how many copies of each prime a number contains.

1

$$72 = 8 \times 9$$
$$\downarrow$$
$$2^3 \times 3^2$$

Why do two different factor trees end alike?

Their terminal prime factors, counted with multiplicity, are unique apart from order.

2

Number	Power of 2	Power of 3
24	3	1
36	2	2

Read the HCF from the exponents.

Take the smaller exponent in each column. HCF = $2^2 \times 3 = 12$.

3

Number	Power of 2	Power of 3
24	3	1
36	2	2

What is the first shared multiple?

Take the larger exponents. LCM = $2^3 \times 3^2 = 72$.

4

$$108 = 2^2 \times 3^3$$
$$\downarrow$$
$$108 \times 3 = 2^2 \times 3^4$$

Which smallest positive integer makes 108 a square when multiplied by it?

Three supplies the missing partner for the odd exponent. The result is 18^2.

$180 = 2^2 \times 3^2 \times 5$
multiply by 5 for a square.

$180 \div 5 = 36$
a square already.

CHAPTER PROBLEM

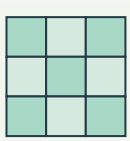
Bells ring every 9 and 12 minutes and ring together now. In how many minutes will they next ring together?

36. LCM(9,12) = 36.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Can the LCM of two positive integers equal the larger number?	Yes, when the smaller divides the larger. For 6 and 24 it is 24.
02 Why is 1 excluded from the primes?	It has only one positive divisor. Allowing extra factors of 1 would also make prime-factor lists non-unique.
03 Find the smallest integer greater than 10 that leaves remainder 3 when divided by both 4 and 6.	15 N – 3 is a multiple of 12; first valid value is 15.
04 A rectangle is 84 cm by 126 cm. Largest square tiles cover it without cutting. What is the tile side? A) 21 cm B) 42 cm C) 84 cm D) 252 cm	42 cm The side divides both dimensions; HCF = 42.
05 A rectangle is 84 cm by 126 cm. How many 42 cm square tiles cover it without cutting?	6 (84/42)×(126/42) = 2×3 = 6.
06 A learner uses HCF(10,15) = 5 for the next coincidence of two 10- and 15-minute cycles. Explain why this fails.	5 is not a multiple of either cycle. The first positive common multiple is 30 minutes.



Why does 9 have an odd number of positive factors?

1 and 9 pair together. The middle factor 3 pairs with itself.

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