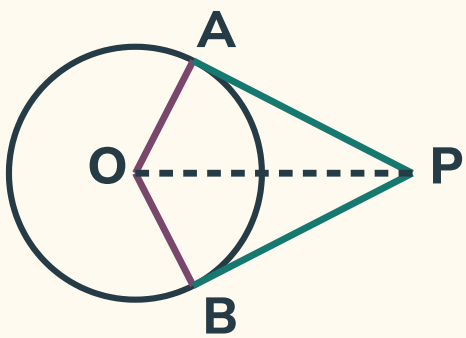
MATHS FIELD GUIDE · GRADE 10

Equal tangents from an external point

OPENING PROBLEM

Why do the two tangent lengths match?



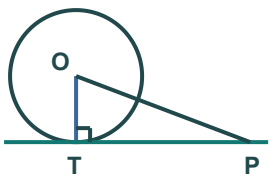
PA = PB

02 Rules and worked cases
03 Trace two tangents from one point
04 Recognise the conditions
05 From the givens to the answer
06 Which relationship proves it?
07 Find the right angle at contact
08 Check your reasoning
09 Check your reasoning

CIRCLE GEOMETRY · ILLUSTRATED PROBLEMS

Equal tangents from an external point

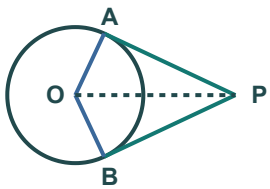
One contact point



OT = 8, OP = 17. Find PT.

The radius meets the tangent at 90°. $PT^2 = 17^2 - 8^2 = 225$, so PT = 15.

Two tangents



PA = 2x + 3 and PB = 17. Find x.

Right triangles OAP and OBP share OP and have equal radii. RHS gives PA = PB, so x = 7.

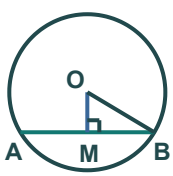
Test a line



A circle has radius 9. Which lines touch it?

Measure perpendicular distance from O. Distance 7 gives two crossings, 9 gives one contact, and 11 misses.

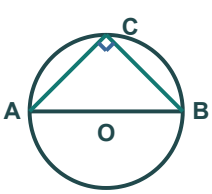
Bisect a chord



Radius 13, OM = 5. Find AB.

The perpendicular from O bisects AB. $MB = \sqrt{(13^2 - 5^2)} = 12$, so AB = 24.

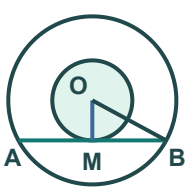
Use a diameter



AB is a diameter. Why is angle ACB 90°?

The intercepted semicircle measures 180°. Its inscribed angle is half that measure.

Use two radii



Radii 10 and 6. Find the outer chord AB.

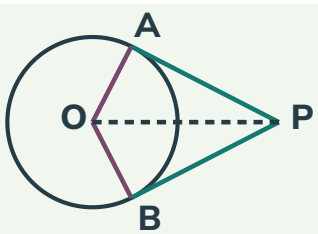
OM is perpendicular to AB at contact with the inner circle. Half-chord $= \sqrt{(10^2 - 6^2)} = 8$. AB = 16.

ILLUSTRATED PROBLEM SHEET

Trace two tangents from one point

Tangents from the same external point to one circle are equal up to their contacts.

1



PA = PB

Why do the two tangent lengths match?

Radii OA and OB are equal. The two right triangles share hypotenuse OP. RHS congruence gives PA = PB.

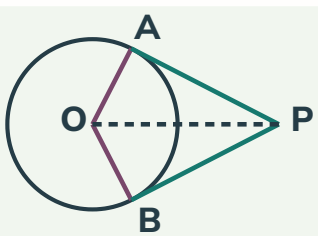
2

$$PA = 2x + 3$$
$$\downarrow$$
$$PB = 5x - 9$$
$$\downarrow$$
$$3x = 12$$

Find x, then the actual tangent length.

x = 4. Both lengths are 11. The value of x and the segment length answer different questions.

3



∠APB = 64°

What are the two smaller angles at P when OP is drawn?

The right triangles are congruent, so OP bisects the angle. Each is 32°.

4

Incircle touch lengths
↓
From A: 2, B: 4, C: 5
↓
Perimeter = 2(2 + 4 + 5)

Why does each number appear twice?

Each vertex sends two equal tangent segments to its adjacent contacts. The perimeter is 22.

OP = 5 and r = 3
each tangent length is 4.

Total of both tangent segments
8.

CHAPTER PROBLEM

PA and PB are tangents from P. PA=12. Find PB.

12. Tangent lengths from one external point are equal.

RULES AND EXAMPLES

Recognise the conditions

Rules, conditions and worked cases

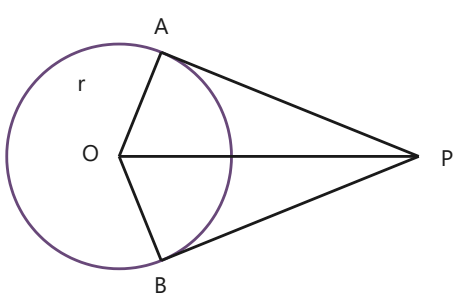
Recognise the case	Mathematical reasoning
Match the right triangles	Radii OA, OB are equal, OP is shared, and both contact angles are 90°. RHS gives PA = PB.
Find an unknown	If PA = 3x + 1 and PB = x + 9, equal lengths give x = 4 and each tangent is 13.
Angle relation	Two right contact angles leave ∠APB + ∠AOB = 180° for the minor central angle.

Read the representations

Two tangents from one external point are equal

Let PA and PB touch a circle with centre O at A and B. Then PA=PB.

This compares tangent segments from the same external point to the same circle, not arbitrary tangent lines.



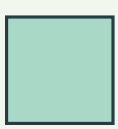
Radius ⊥ tangent at contact

Prove equality with right-triangle congruence

OA⊥PA and OB⊥PB by the radius theorem. OA=OB are radii and OP is common.

The right triangles OAP and OBP are congruent by RHS. Therefore PA=PB; the corresponding angles also show OP bisects ∠APB.

1. **Right angles**Radius ⊥ tangent
2. **Equal hypotenuse**OP is common
3. **Equal leg**OA=OB (radii)
4. **RHS congruence**PA=PB



square

If a quadrilateral is a square, are its diagonals equal?

Yes. They are hypotenuses of congruent right triangles.

WORKED EXAMPLES

From the givens to the answer

Use equal tangent lengths in an equation

1. PA=2x+3 and PB=5x−6 from the same external P.
2. 2x+3=5x−6, giving x=3 and PA=PB=9.
3. Check both lengths are positive before interpreting.

Find a missing tangent length

Tangents PA and PB touch a circle at A and B. PA = 3x − 1 and PB = x + 9. Find each length.

1. Tangents from P are equal, so 3x − 1 = x + 9. Hence x = 5, and both lengths are 14. Equality applies to the segments ending at the points of contact.

Check another case

Example 1

Tangents PA=x+2 and PB=2x−8. Find x.

10

Equate lengths and solve x+2=2x−a.

Example 2

A circle has radius 27, centre distance OP=45. Find either tangent length from P.

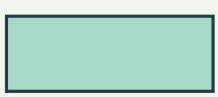
36

At contact T, OT is perpendicular to PT. $PT^2 = 45^2 - 27^2 = 1{,}296$, so PT = 36.

Trace the perimeter with shared tangents

An incircle touches the sides of triangle ABC. Tangent segments from A, B and C have lengths 3, 4 and 5 cm respectively. Then AB = 7, BC = 9 and CA = 8 cm.

The perimeter is 24 cm. Each vertex contributes its length twice, giving the same result as 2(3 + 4 + 5).



rectangle

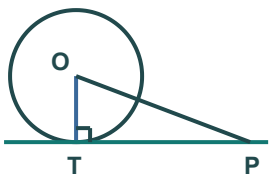
If the diagonals of a quadrilateral are equal, must it be a square?

A non-square rectangle is a counterexample. Equal diagonals alone do not force equal sides.

CIRCLE GEOMETRY · PROOF AND RECONSTRUCTION

Which relationship proves it?

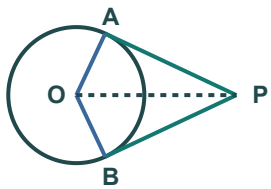
Prove perpendicularity



Why must the radius to T meet the tangent at a right angle?

If the perpendicular foot H were elsewhere on the tangent, OH would be shorter than OT. H would be inside the circle, making the line cross it twice.

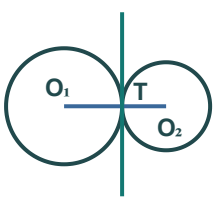
Bisect an external angle



Why does OP split angle APB equally?

OAP and OBP are right triangles with equal radii and common hypotenuse OP. RHS congruence gives equal angles at P.

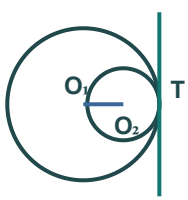
Touching externally



Radii are 7 cm and 4 cm. Find the distance between the centres.

Both radii are perpendicular to the common tangent at T, so they lie on one straight line. T is between the centres. Add: 7 + 4 = 11 cm.

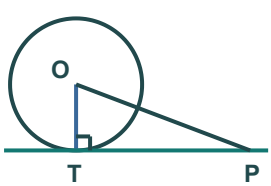
Touching internally



The same circles now touch internally. What is the centre distance?

Both radii run from T to the same side of the tangent. The distance between centres is the excess of the longer radius: 7 − 4 = 3 cm.

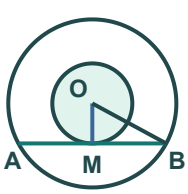
Recover the centre distance



A tangent segment is 20 cm. The radius is 15 cm. Is the centre 35 cm from P?

OP is the hypotenuse, so $OP^2 = 20^2 + 15^2 = 625$ and OP = 25 cm. The sum 35 cm follows the broken path P–T–O.

Reconstruct a radius



The outer radius is 10 cm. A 12 cm outer chord touches the inner circle. What is the inner radius?

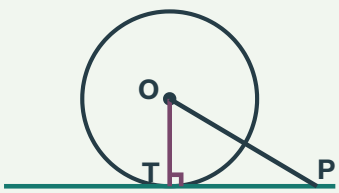
Contact M bisects the chord. The right triangle has hypotenuse 10 and one leg 6. The other leg OM = $\sqrt{(100 - 36)} = 8$ cm.

ILLUSTRATED PROBLEM SHEET

Find the right angle at contact

At the point of tangency the radius is perpendicular to the tangent line.

1

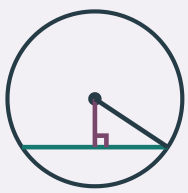


OT ⊥ PT

OT = 5 cm and OP = 13 cm. How long is PT?

OP is the hypotenuse. PT = √(169 − 25) = 12 cm.

2



r = 5, d = 3

How can distance to a line distinguish a secant from a tangent?

Three is less than the radius five. The line crosses twice. Half the chord is four.

3

PA = PB

↓

3x − 2 = x + 8

↓

x = 5

Tangents from one external point have these expressions. Find their lengths.

Both equal 13. They run from the same point to different contacts on the same circle.

4

∠OAP = ∠OBP = 90°

↓

∠APB = 54°

↓

∠AOB = 126°

Why do the last two angles sum to 180°?

The right angles already use 180° of quadrilateral OAPB's 360°.

Centre distance to line below r
two intersections.

Centre distance to line above r
no intersections.

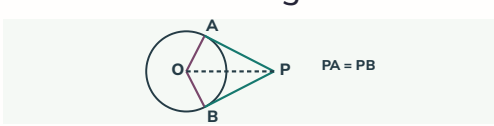
CHAPTER PROBLEM

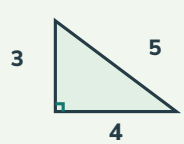
Tangents PA=x+2 and PB=2x−5. Find x.

7. Equate lengths and solve x+2=2x−a.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Tangents from different external points measure 8 and 11. Is that a contradiction?	No. The equality theorem compares two tangents from the same external point.
02 Can a point inside a circle have a tangent segment to the circle?	No. Its centre distance is less than the radius. Every line through it intersects the circle twice.
03 A circle has radius 21, centre distance OP=35. Find either tangent length from P.	28 Let T be the contact point. Radius OT is perpendicular to PT, so $PT^2 = 35^2 - 21^2 = 784$. The positive root gives 28.
04 Which congruence establishes equal tangents? A) SSA for any two triangles B) RHS using OP and equal radii C) Equal areas alone D) One equal angle	RHS using OP and equal radii The triangles have a right angle, common hypotenuse and equal radius-leg.
05 If ∠APB=64° for two tangents, find ∠APO in degrees.	32° Congruence shows OP bisects the angle at P.
06 If tangents PA,PB make ∠APB=70°, find minor ∠AOB in degrees. 	110° Quadrilateral OAPB has two right angles; the remaining angles sum to 180°.

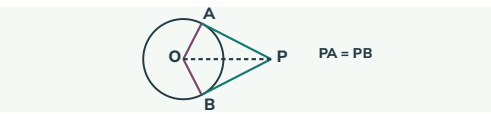
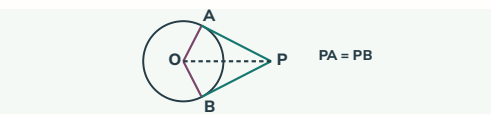
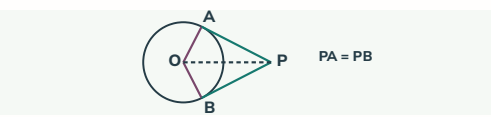


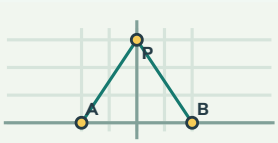
Use one diagram to distinguish a theorem from its converse.

A right angle implies $3^2 + 4^2 = 5^2$ for these lengths. The equality also proves the angle is right by the converse of Pythagoras.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Prove PA=PB for tangents from external point P. 	Join OA,OB,OP; identify right angles at A,B. Use OA=OB and common OP for RHS congruence. Conclude corresponding tangent lengths are equal.
08 Can tangents from two different external points be assumed equal?	No. The theorem uses one shared external point. For a fixed radius r, tangent length $\sqrt{(OP^2-r^2)}$ varies with OP.
09 PA and PB are tangents from P. PA=15. Find PB. 	15 Tangent lengths from one external point are equal.
10 Tangents PA=x+2 and PB=2x−6. Find x. 	8 Tangents from the same external point are equal. Set x + 2 = 2x − 6 and subtract x from both sides. Then x = 8, giving PA = PB = 10.
11 PA and PB are tangents from P. PA=21. Find PB.	21 Tangent lengths from one external point are equal.



Why is P(0, 3) equidistant from A(−2, 0) and B(2, 0)?

The perpendicular bisector is the y-axis. Both squared distances equal 4 + 9 = 13.

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