MATHS FIELD GUIDE · GRADE 10

# Discriminant and nature of roots

OPENING PROBLEM

## What stays fixed across the family?

| k | 3 | 4 | 5 |
|---|---|---|---|
| D for $x^2 - 4x + k$ | 4 | 0 | −4 |
| Real zeros | 2 | 1 | 0 |

02 Watch two zeros meet and disappear
03 Rules and worked cases
04 From the givens to the answer
05 Choose a quadratic method
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

# Watch two zeros meet and disappear

For a real quadratic with $a \neq 0$, the sign of $b^2 - 4ac$ determines the number of distinct real zeros.

1

$$x^2 - 4x + 3$$
$$\downarrow$$
$$D = 16 - 12 = 4$$
$$\downarrow$$
$$x = 1, 3$$

### How does factoring check the positive discriminant?

$(x - 1)(x - 3)$ has two distinct zeros.

2

$$x^2 - 4x + 4$$
$$\downarrow$$
$$D = 0$$
$$\downarrow$$
$$(x - 2)^2$$

### Why is just one x-value present?

The factor $x - 2$ occurs twice. Both factors vanish at the same input, $x = 2$.

3

$$x^2 - 4x + 5$$
$$\downarrow$$
$$D = -4$$
$$\downarrow$$
$$(x - 2)^2 + 1$$

### Why can this graph miss the horizontal axis?

Every real square is nonnegative, so its output is at least one.

4

| k | 3 | 4 | 5 |
|---|---|---|---|
| D for $x^2 - 4x + k$ | 4 | 0 | −4 |
| Real zeros | 2 | 1 | 0 |

### What stays fixed across the family?

The axis $x = 2$ stays fixed. Increasing k raises the entire curve.

**$D = 1$**
**two real roots.**

**$D = -1$**
**no real roots.**

CHAPTER PROBLEM

### Find D for $x^2-8x+16=0$.

0. $D = (-8)^2 - 4 \times 1 \times 16 = 64 - 64 = 0$. The two real roots coincide.

MATHS REFERENCE · GRADE 10

# Discriminant and nature of roots

## Rules, conditions and worked cases

| Recognise the case | Mathematical reasoning |
|---|---|
| **D > 0** | $x^2 - 3x + 1$ has D = 5, hence two distinct real roots. The irrational square root keeps them irrational. |
| **D = 0** | $4x^2 - 12x + 9 = (2x - 3)^2$ has one repeated root 3/2. |
| **D < 0** | $x^2 + 2x + 5$ has D = −16. No real x makes $(x + 1)^2 + 4$ zero. |

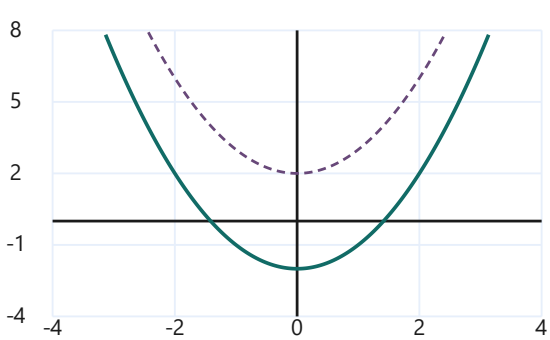
## Read the representations

### The discriminant counts real intercepts

For $ax^2+bx+c$ with $a \neq 0$, $D=b^2-4ac$ is the radicand in the quadratic formula. $D>0$ gives two distinct real roots, $D=0$ one repeated real root and $D<0$ no real roots.

These correspond to two crossings, one tangency or no intersection with the horizontal axis. The sign of a controls opening direction, not the root count by itself.

Different vertical positions change the intercept count




$y=x^2-2$ crosses twice; $y=x^2+2$ never reaches zero.

### Keep the equation genuinely quadratic

If a parameter makes $a=0$, reclassify the resulting equation before applying a quadratic root-count rule.

A negative discriminant rules out real roots; this course does not require treating complex roots as real intercepts.

**$(x - 2)(x + 3)$**

↓

**$x^2 + x - 6$**

### Which inputs give zero?

2 and −3. Each makes a different factor vanish.

WORKED EXAMPLES

# From the givens to the answer

### A repeated root

1. For $x^2-6x+9$, $D=36-36=0$.
2. The polynomial is $(x-3)^2$, so the repeated root is 3.

### No real crossing

1. For $x^2+2x+5$, $D=4-20=-16$.
2. Equivalently $(x+1)^2+4$ stays positive.

### Does a negative discriminant mean a negative root?

1. It means the formula would require a square root of a negative number.
2. No real roots exist; the claim concerns root existence rather than root sign.

## Check another case

### Example 1

How many distinct real roots has $(x-8)^2=4$?

2

A positive square value gives two branches.

### Example 2

For $x^2-18x+82=0$, which describes real roots?

No real roots · Two distinct positive roots · One repeated root · Every real number

No real roots

$D=-4$.

### Move from two roots to none

In $x^2 - 6x + k = 0$ the discriminant is $36 - 4k$. At $k = 9$ there is one repeated root, 3.

For $k < 9$ it is positive, giving two distinct real roots. For $k > 9$ it is negative, giving no real roots.

| x | −3 | 0 | 2 |
|---|---|---|---|
| p(x) | 0 | −6 | 0 |

### Which table entry is the vertical intercept?

At $x = 0$ the output is $-6$. The two zero outputs locate horizontal intercepts.

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# Choose a quadratic method

First write the equation with zero on one side. Factor, complete the square, or use the formula.

1

$$6x^2 + x - 2 = 0$$
$$\downarrow$$
$$(3x + 2)(2x - 1) = 0$$
$$\downarrow$$
$$x = -2/3 \text{ or } 1/2$$

### Which products make the middle term?

The cross-products are −3x and +4x, leaving +x.

2

$$x^2 - 6x + 5 = 0$$
$$\downarrow$$
$$(x - 3)^2 = 4$$
$$\downarrow$$
$$x = 1 \text{ or } 5$$

### Why do two roots appear?

Both 2 and −2 square to 4, so x − 3 can take either sign.

3

$$x^2 - 2x - 1 = 0$$
$$\downarrow$$
$$D = 8$$
$$\downarrow$$
$$x = 1 \pm \sqrt{2}$$

### When is an exact radical useful?

The discriminant is positive and is not a perfect square. The formula preserves both exact roots.

4

$$w(w + 3) = 40$$
$$\downarrow$$
$$(w - 5)(w + 8) = 0$$
$$\downarrow$$
$$w = 5$$

### Which algebraic answer fails the model?

A side length cannot be −8. The rectangle is 5 by 8.

$x^2 - 9 = 0$
**$x = \pm 3$.**

$x^2 + 9 = 0$
**no real roots.**

CHAPTER PROBLEM

### How many distinct real roots has $(x-5)^2=4$?

2. A positive square value gives two branches.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **01**<br>If a discriminant is positive, must both roots be integers? | No. For $x^2 - 2 = 0$ it is 8. The two real roots are $\pm\sqrt{2}$. |
| **02**<br>What happens to $x^2 - 6x + k = 0$ at $k = 9$? | It becomes $(x - 3)^2 = 0$. The two root values coincide at 3. |
| **03**<br>For $x^2-14x+50=0$, which describes real roots?<br>A) No real roots B) Two distinct positive roots<br>C) One repeated root D) Every real number | No real roots<br>The discriminant is $(-14)^2 - 4 \times 1 \times 50 = -4$. Equivalently, $(x - 7)^2 + 1$ is positive for every real x. |
| **04**<br>If the leading coefficient becomes zero for a parameter value 8, what should happen first?<br>A) Reclassify the original equation<br>B) Still divide by 2a C) Assume two roots<br>D) Ignore the parameter | Reclassify the original equation<br>The quadratic formula assumes a nonzero leading coefficient. |
| **05**<br>For $y=(x-3)^2$, find its repeated root. | 3<br>Write the square as $(x - 3)(x - 3)$. Each factor vanishes at $x = 3$, so the same root occurs twice. |
| **06**<br>Connect D=0 with the graph $y=(x-4)^2$. | The square is nonnegative and zero only at 4.<br>The vertex lies on the axis.<br>One distinct real root is repeated algebraically. |

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

### Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **07**<br>Explain why $y=-(x-5)^2-1$ also has no real x-intercepts. | The negative square is at most zero.<br>Subtracting one makes the value strictly negative.<br>Opening downward does not itself guarantee real roots. |
| **08**<br>Find D for $x^2-10x+25=0$. | 0<br>The terms $b^2$ and $4ac$ agree. |
| **09**<br>How many distinct real roots has $(x-6)^2=4$? | 2<br>A positive square value gives two branches. |
| **10**<br>Find D for $x^2-18x+78=0$. | 12<br>$D = (-18)^2 - 4 \times 1 \times 78 = 324 - 312 = 12$. The two real roots are distinct. |
| **11**<br>Find D for $x^2-14x+49=0$. | 0<br>The terms $b^2$ and $4ac$ agree. |

**Sum = −1**
↓
**Product = −6**
↓
**$x^2 + x - 6$**

**Reconstruct a monic quadratic from these two root facts.**

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are $(x - 2)$ and $(x + 3)$.

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