MATHS FIELD GUIDE · GRADE 10

Deriving the sum of an AP

OPENING PROBLEM

Find the sum of the forward row.

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

02 Pair the ends to find a sum
03 Rules and worked cases
04 From the givens to the answer
05 Work back from an arithmetic total
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Pair the ends to find a sum

Forward and reverse copies of an AP have equal column sums. Divide their combined total by two.

1

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

Find the sum of the forward row.

Four pairs give 52, twice the sum. The sum is 26.

2

$$1 + 2 + \cdots + 20$$
$$\downarrow$$
$$20 \times 21 \div 2$$
$$\downarrow$$
$$210$$

Which two terms form the first pair?

1 and 20. Every pair of opposite positions totals 21.

3

$$4 + 7 + 10 + 13 + 16$$
$$\downarrow$$
$$2 \times 20 + 10$$
$$\downarrow$$
$$50$$

What if one middle term is left over?

Add the two outer pairs and then the middle 10. The average is 10.

4

$$S_{10} = 190$$
$$\downarrow$$
$$S_9 = 153$$
$$\downarrow$$
$$a_{10} = 37$$

How can two totals reveal just one term?

The first nine terms occur in both totals. Subtraction leaves the tenth alone.

Five equally spaced terms
total = five times the middle term.

Six equally spaced terms
total = three times the sum of the endpoints.

CHAPTER PROBLEM

Find S_{10} for AP a=4,d=2.

130. The tenth term is 4 + 9 × 2 = 22. Pair the first and last terms: 10 × (4 + 22)/2 = 130.

MATHS REFERENCE · GRADE 10

Deriving the sum of an AP

Rules, conditions and worked cases

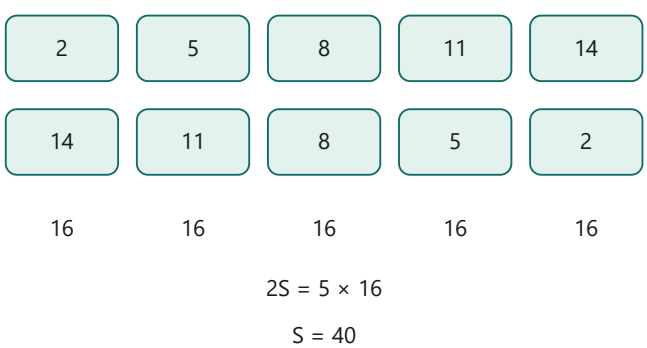
Recognise the case	Mathematical reasoning
Reverse and add	Write an AP forward and backward. Each paired column totals a + l, giving 2S = n(a + l).
Numerical check	4 + 7 + 10 + 13 + 16 has mean (4 + 16)/2 = 10. Five terms total 50.
When l is unknown	Substitute l = a + (n − 1)d into the pair formula: S = n[2a + (n − 1)d]/2.

Read the representations

Pair a list with its reverse

Write $S=a+(a+d)+\ldots+l$. Reverse the list below it.

Every vertical pair equals a+l, and there are n pairs. Thus $2S=n(a+l)$ and $S_n=n[2a+(n-1)d]/2$.



A term and a total answer different questions

a_n is the value at one position; S_n adds the first n terms.

$S_n-S_{n-1}=a_n$. In a context, identify whether the question asks for one stage or all stages together.

7 **11** **15** **19** **23**

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

WORKED EXAMPLES

From the givens to the answer

Add 6+10+14+...+42

1. The common difference is 4. Solve $42=6+(n-1)4$ to obtain $n=10$.
2. $S=10(6+42)/2=240$.
3. Pairing the first and last gives 48; there are five such pairs.

Pair an odd number of terms

Find 4 + 7 + 10 + 13 + 16 + 19 + 22 without adding from left to right.

1. The outside pairs each sum to 26. Three pairs contribute 78. Add the unpaired middle term 13 to get 91. The average is exactly 13.

Check another case

Example 1

An AP has 12 terms, first 8, last 41. Find its sum.

294

Use $n(a+l)/2$.

Example 2

For $a=9, d=2$, find a_8.

23

This asks for one term, not S8.

Why does the backwards row help?

2	5	8	11
11	8	5	2

Write 2 + 5 + 8 + 11, then underneath write 11 + 8 + 5 + 2. Every column totals 13. Four columns total 52.

The two rows contain two copies of the original sum. One copy is 26. The same argument works with any number of terms.

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

ILLUSTRATED PROBLEM SHEET

Work back from an arithmetic total

Use $S_n = n(a_1 + a_n)/2$. A valid term count is a positive integer.

1

Rows 4, 8, 12, ...
↓
$S_n = 2n(n + 1)$
↓
Total 84

How many rows were used?

$n(n + 1) = 42$. Six times seven gives 42, hence six rows. Check 4 + 8 + 12 + 16 + 20 + 24 = 84.

2

$S_7 = 91$
↓
$S_6 = 69$
↓
$a_7 = 22$

Why can subtraction recover the next term?

The first six appear in both totals. Only the seventh remains.

3

10 terms
↓
First 3, total 210
↓
3 + last = 42

Find the last term.

The mean term is 21. The two endpoints sum to 42, so the last is 39. Nine gaps give d = 4.

4

2 4 6 8 10

Can the sum of the first n positive even numbers be 25 for any whole number n?

The first four sum to 20. The first five sum to 30. All terms are positive, so 25 is skipped.

$S_5 = 40$ and $S_4 = 29$
term 5 is 11.

$a_5 = 11$ and $S_4 = 29$
$S_5 = 40$.

CHAPTER PROBLEM

An AP has 12 terms, first 5, last 38. Find its sum.

258. Use n(a+l)/2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 The first n natural numbers sum to 55. Find n.	$10 \times 11 \div 2 = 55$. The sums strictly increase for positive n, so $n = 10$.
02 Why reject a negative solution for n even if it satisfies a quadratic for the sum?	A negative count does not identify any initial group of terms in the sequence.
03 For $a=7, d=2$, find a_8.	21 This asks for one term, not S8.
04 Why is there a factor 1/2 in $S=n(a+l)/2$? A) Only half the terms count. B) Adding the forward and reverse lists counts the sum twice. C) Every AP has even length. D) The difference is always 2.	Adding the forward and reverse lists counts the sum twice. The pairing argument produces 2S.
05 $S_9=109$ and $S_8=90$. Find a_9.	19 Subtract consecutive partial sums.
06 Find the sum of the first 3 positive odd integers.	9 $1 + 3 + 5 = 9$. The first n positive odd integers sum to n^2.

$$7 + 4(n - 1) = 39$$
$$\downarrow$$
$$4(n - 1) = 32$$
$$\downarrow$$
$$n = 9$$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Derive the sum formula when n is odd.	Use the forward-plus-reverse lists rather than trying to split the original list into pairs. There are n equal pairs across the two lists, so divide the result by 2.
08 If $a_n=3n+2$, explain why S_n is quadratic in n.	Substitute a=5,d=3 in the sum formula. $S_n=n(3n+7)/2$ contains a nonzero n^2 term.
09 Find S_{10} for AP a=5,d=2.	140 S10=10[2a+18]/2.
10 An AP has 12 terms, first 6, last 39. Find its sum.	270 Use n(a+l)/2.
11 Find S_{10} for AP a=7,d=2.	160 The tenth term is 7 + 9 × 2 = 25. Pair the first and last terms: each of five pairs sums to 32, for a total of 160.

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

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