MATHS FIELD GUIDE · GRADE 10

Constructing a polynomial and parameter problems

OPENING PROBLEM

Write the monic polynomial for the second row.

Root 1	Root 2	Sum	Product
2	5	7	10
−2	5	3	−10

02 Recover coefficients from two zeros
03 Rules and worked cases
04 From the givens to the answer
05 Connect factors, roots and the constant term
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Recover coefficients from two zeros

For $ax^2 + bx + c$ with $a \neq 0$, the two zeros (counting repeats) sum to $-b/a$ and multiply to c/a.

1

Root 1	Root 2	Sum	Product
2	5	7	10
−2	5	3	−10

Write the monic polynomial for the second row.

$x^2 - 3x - 10$. The sign of the middle coefficient is the opposite of the sum.

2

Zeros −1 and 4
↓
$(x + 1)(x - 4)$
↓
$x^2 - 3x - 4$

What if the leading coefficient must be 3?

Triple the whole polynomial to obtain $3x^2 - 9x - 12$. Its zeros stay fixed.

3

$2x^2 - 5x + c$
↓
One zero = 2
↓
$8 - 10 + c = 0$

Find c and the second zero.

c = 2. The sum is 5/2, so the other zero is 1/2. Their product is 1.

4

Sum = 0
↓
Product = −9
↓
$x^2 - 9$

Can opposite zeros make the linear coefficient vanish?

Yes. The zeros 3 and −3 add to zero.

Roots 0 and 7
monic polynomial $x^2 - 7x$.

Roots −7 and 7
monic polynomial $x^2 - 49$.

CHAPTER PROBLEM

Construct the monic polynomial with zeros 4 and −1. Give its constant coefficient.

-4. The two factors are $(x - 4)$ and $(x + 1)$. Their product is $x^2 - 3x - 4$, so the constant coefficient is −4.

MATHS REFERENCE · GRADE 10

Constructing a polynomial and parameter problems

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Build from roots	Zeros 2 and −5 give $(x - 2)(x + 5) = x^2 + 3x - 10$.
Build from sum and product	Sum 7 and product 10 give the monic $x^2 - 7x + 10$. Factor to check zeros 2 and 5.
Determine a parameter	If 3 is a zero of $x^2 + kx - 6$, then $9 + 3k - 6 = 0$, giving $k = -1$.

Read the representations

Choose zeros first, then expand

Zeros α and β give the monic quadratic $(x-\alpha)(x-\beta)$. Any nonzero constant multiple has the same zeros.

Use x^2-Sx+P when the sum S and product P are known. A specified leading coefficient fixes the multiplier.

From information to form

Given	Quadratic
Zeros 2 and −3	$(x-2)(x+3)=x^2+x-6$
Sum 5, product 4	x^2-5x+4
Zeros 2 and −3, leading coefficient 3	$3x^2+3x-18$

A root condition is an equation in a parameter

If 2 is a zero of x^2+kx-6, then $4+2k-6=0$.

Solve for k, then check the original polynomial. Keep the leading coefficient nonzero.

×	a	b
a	a^2	ab
b	ab	b^2

Where does 2ab in $(a + b)^2$ come from?

The two rectangular off-diagonal products are both ab.

WORKED EXAMPLES

From the givens to the answer

Construct from sum and product

1. For sum −1 and product −12, choose x^2+x-12.
2. Factor $(x+4)(x-3)$; zeros −4 and 3 have sum −1 and product −12.

Use a supplied zero to find a coefficient

1. If 3 is a zero of $2x^2+kx-3$, substitute: $18+3k-3=0$.
2. Thus $k=-5$. Check $2(9)-5(3)-3=0$.

Choose a polynomial from two zeros

Which quadratic with leading coefficient 2 has zeros −3 and 4?

1. Use one factor per zero. Expand $2(x + 3)(x - 4)$ to get $2x^2 - 2x - 24$. Both −3 and 4 make a factor zero.

Check another case

Example 1

A monic quadratic has zero sum 10. Give its coefficient of x.

-10

The x-coefficient is minus the sum.

Example 2

If 2 is a zero of $x^2+kx-22$, find k.

9

Substitute $x = 2$: $4 + 2k - 22 = 0$, so $2k = 18$ and $k = 9$.

Find a coefficient from a prescribed root

A root of $x^2 + kx - 12$ is 3. Substituting gives $9 + 3k - 12 = 0$, so $k = 1$.

Factor the resulting polynomial as $(x - 3)(x + 4)$. The second root is −4 and their product is −12.

$$(3x - 2)^2$$
$$\downarrow$$
$$(3x)^2 - 2(3x)(2) + 2^2$$
$$\downarrow$$
$$9x^2 - 12x + 4$$

Which signs change in a squared difference?

The middle product is negative. The constant is positive because $(-2)^2 = 4$.

ILLUSTRATED PROBLEM SHEET

Connect factors, roots and the constant term

A product is zero when at least one factor is zero.

1

$$(x - 2)(x + 3)$$
$$\downarrow$$
$$x^2 + x - 6$$

Which inputs give zero?

2 and −3. Each makes a different factor vanish.

2

x	−3	0	2
p(x)	0	−6	0

Which table entry is the vertical intercept?

At x = 0 the output is −6. The two zero outputs locate horizontal intercepts.

3

$$2(x - 2)(x + 3)$$
$$\downarrow$$
$$2x^2 + 2x - 12$$

Does doubling the polynomial change its roots?

No. A nonzero constant factor changes the outputs but preserves the zeros.

4

$$\text{Sum} = -1$$
$$\downarrow$$
$$\text{Product} = -6$$
$$\downarrow$$
$$x^2 + x - 6$$

Reconstruct a monic quadratic from these two root facts.

Use $x^2 - (\text{sum})x + \text{product}$. Its factors are (x − 2) and (x + 3).

$p(x) = (x - 4)^2$
one distinct zero.

$p(x) = (x - 4)(x + 4)$
two distinct zeros.

CHAPTER PROBLEM

A monic quadratic has zero sum 7. Give its coefficient of x.

-7. The x-coefficient is minus the sum.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 A monic quadratic has zeros that add to 6 and multiply to 9. How many distinct zeros?	$x^2 - 6x + 9 = (x - 3)^2$, so one distinct zero.
02 Can a nonzero quadratic have three distinct real zeros?	No. Three distinct linear factors would require degree at least three.
03 If 2 is a zero of $x^2+kx-18$, find k.	7 Substitute x = 2: 4 + 2k − 18 = 0, so 2k = 14 and k = 7.
04 Which has zeros 8 and 3? A) $(x-8)(x-3)$ B) $(x+8)(x+3)$ C) $x^2+11x+24$ D) $x^2-11x-24$	$(x-8)(x-3)$ Each desired zero makes one factor vanish.
05 Zeros of $2x^2+kx+8$ have sum 9. Find k.	-18 $-k/2=a$.
06 Zeros of x^2-4x+k are reciprocal. Find k.	1 Reciprocal zeros have product 1.

$49x^2 - 16$
↓
$(7x)^2 - 4^2$
↓
$(7x - 4)(7x + 4)$

Why is there no middle term?

The cross-products +28x and −28x cancel.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Write two different quadratics with zeros 0 and 4.	x(x−4) and 2x(x−4) both work. Multiplication by a nonzero constant preserves all zeros.
08 A learner uses x^2+Sx+P for specified sum S and product P. Repair the sign.	Expansion gives $x^2-(\alpha+\beta)x+\alpha\beta$. Use x^2-Sx+P and check both relationships.
09 Construct the monic polynomial with zeros 5 and −1. Give its constant coefficient.	-5 Use one factor per zero: $(x-5)(x+1) = x^2 - 4x - 5$. The constant coefficient is −5.
10 The two zeros of a monic quadratic add to 8. Give its coefficient of x.	-8 In $x^2 + bx + c$, the zeros add to −b. If their sum is 8, then b = −8.
11 Construct the monic polynomial with zeros 7 and −1. Give its constant coefficient.	-7 The factors are (x − 7) and (x + 1). Their product is $x^2 - 6x - 7$, so its constant coefficient is −7.

×	x	−3
x	x^2	−3x
5	5x	−15

Collect the products of (x + 5)(x − 3).

The middle terms combine to 2x, giving $x^2 + 2x - 15$.

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