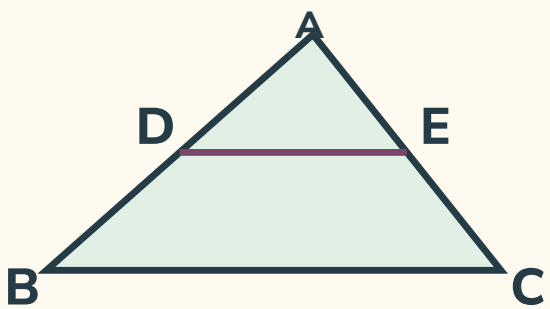
MATHS FIELD GUIDE · GRADE 10

Basic Proportionality Theorem proof

OPENING PROBLEM

Use AB = 8, AC = 6. AD = 4. Find AE.



DE ∥ BC

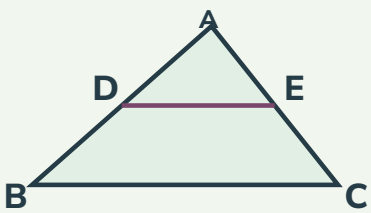
02 A parallel cut creates two similar triangles
03 Rules and worked cases
04 From the givens to the answer
05 Choose a base and its perpendicular height
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

A parallel cut creates two similar triangles

D lies on AB, E lies on AC and DE is parallel to BC unless the question asks you to test it.

1



DE ∥ BC

Use AB = 8, AC = 6. AD = 4. Find AE.

D is halfway to B. The smaller triangle is at half scale, so AE = 3.

2

AD = 3, DB = 6
↓
AE = 4
↓
AD/DB = AE/EC

Recover the missing segment EC.

The lower piece is twice the upper piece on AB. EC must be twice 4, which is 8.

3

Side	Upper	Lower
AB	3	5
AC	6	10

If parallelism was not given, can you deduce it?

Yes. 3/5 = 6/10. The converse applies because D and E are internal points.

4

AD/AB = 2/5
↓
Area ADE / Area ABC = 4/25

What fraction of the large area remains below DE?

1 − 4/25 = 21/25. The area factor is the square of the length factor.

AD:DB = 1:3
AD:AB = 1:4.

AD:AB = 3:4
AD:DB = 3:1.

CHAPTER PROBLEM

Prove BPT for a parallel line DE inside ΔABC.

Use [ADE]/[BDE]=AD/DB and [ADE]/[CDE]=AE/EC. Justify [BDE]=[CDE] by common base DE and same parallels. Conclude the segment ratios are equal.

MATHS REFERENCE · GRADE 10

Basic Proportionality Theorem proof

Rules, conditions and worked cases

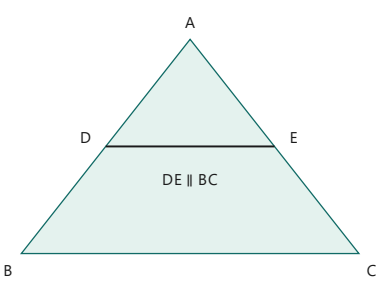
Recognise the case	Mathematical reasoning
Setup	In ΔABC, D and E lie inside AB and AC. If DE ∥ BC, then AD/DB = AE/EC.
Why ratios match	Triangles ADE and BDE share altitude from E to AB. ADE and CDE share altitude from D to AC. BDE and CDE have equal area on base DE between parallels.
Apply the ratio	AD = 4, DB = 6, AE = 10 give 4/6 = 10/EC. Hence EC = 15; whole-side ratios also agree.

Read the representations

Parallel cuts divide the two sides proportionally

In triangle ABC, D lies on AB and E on AC, with DE parallel to BC. The Basic Proportionality Theorem states AD/DB=AE/EC.

The statement compares each upper segment to its remaining lower segment. AD/AB=AE/AC is an equivalent whole-side form, not the same pair of denominators.



D lies on AB; E lies on AC.

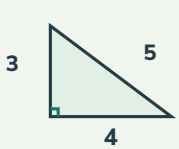
Prove the ratio using areas

Triangles ADE and BDE share an altitude from E to AB, so their areas have ratio AD:DB.

Triangles ADE and CDE share an altitude from D to AC, so their areas have ratio AE:EC.

Triangles BDE and CDE share base DE and lie between the same parallels DE and BC, so their areas are equal. Equate the two ratios.

1. **Same altitude**[ADE]/[BDE] = AD/DB
2. **Same altitude**[ADE]/[CDE] = AE/EC
3. **Same base and parallels**[BDE] = [CDE]
4. **Combine**AD/DB = AE/EC



Scale every side by two. Which lengths result?

6, 8 and 10. The angles stay fixed, and the perimeter doubles.

WORKED EXAMPLES

From the givens to the answer

Apply the theorem with segment lengths

1. DE∥BC, AD=4, DB=6, AE=5. Then 4/6=5/EC.
2. Cross-multiply: 4EC=30, so EC=7.5.
3. The whole sides are AB=10, AC=12.5; 4/10=5/12.5=0.4.

Explain why two areas match

In triangle ABC, D lies on AB and E on AC, with DE parallel to BC. Why do BDE and CDE have equal areas?

1. Both use base DE. B and C lie on the same parallel line, so their perpendicular heights to DE are equal. Each area is half of the same base times the same height.

Check another case

Example 1

point arrangement

Point	D	E
Location	Between A and B	Between A and C

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

DE∥BC with D∈AB,E∈AC. AD=3,AB=9,AC=24. Find AE.

8

The whole-side fraction is 1/3.

Example 2

point arrangement

Point	D	E
Location	Between A and B	Between A and C

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

In the area proof, [ADE]=4,[BDE]=8. Find AD/DB.

0.5

Shared altitude gives base ratio equal to area ratio.

Length	Small	Large
AB ↔ PQ	4	10
BC ↔ QR	6	?

If ABC and PQR are similar in that order, find QR.

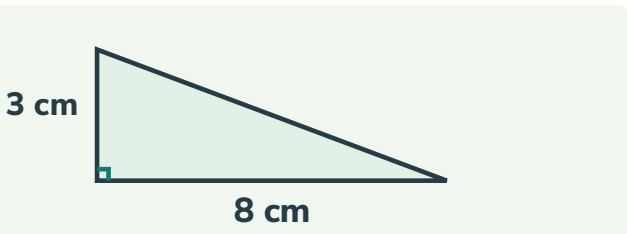
The length scale is 10/4 = 2.5, so QR = 15.

ILLUSTRATED PROBLEM SHEET

Choose a base and its perpendicular height

Sliding the apex along a line parallel to the base preserves the area.

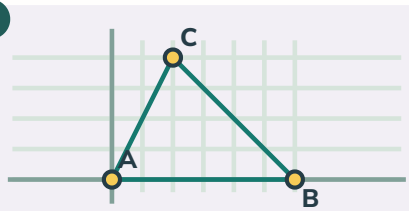
1



What is the area of this right triangle?

Half of an 8 by 3 rectangle, or 12 cm².

2

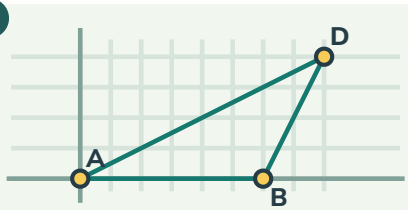


A (0, 0)
B (6, 0)
C (2, 4)

For A(0,0), B(6,0), C(2,4), which height belongs to AB?

The vertical distance is 4. Area = ½ × 6 × 4 = 12 square units.

3



A (0, 0)
B (6, 0)
D (8, 4)

The new apex is D(8,4). Does the area change?

No. The perpendicular foot falls outside AB, but the height to its line is still 4.

4

Area = 30 cm²
↓
Base = 10 cm
↓
Height = 2 × 30 ÷ 10

Find the height without knowing the sloping sides.

The required perpendicular height is 6 cm.

Base doubles, height fixed
area doubles.

Base doubles, height halves
area stays fixed.

CHAPTER PROBLEM

Why is AD/DB=AE/AC usually wrong?

The first denominator is a remaining segment, while the second is a whole side. Use AD/DB=AE/EC or AD/AB=AE/AC.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

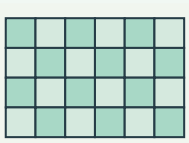
Data A

point arrangement

Point	D	E
Location	Between A and B	Between A and C

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

Try the problem	Solution and reasoning
01 · Data A In ΔABC, $D \in AB, E \in AC, DE \parallel BC$. $AD=4, DB=8, AE=6$. Find EC.	12 $AD/DB = 1/2 = 6/EC$.
02 · Data A $DE \parallel BC$ with $D \in AB, E \in AC$. $AD=3, AB=9, AC=15$. Find AE.	5 The whole-side fraction is 1/3.
03 · Data A $DE \parallel BC$, $AD=4, DB=6$. Find AD/AB.	0.4 $AB=10$, not 6.
04 Which equality is valid under the Basic Proportionality Theorem? A) $AD/DB=AE/EC$ B) $AD/DB=AC/AE$ C) $AB/DB=AE/AC$ D) $AD/AE=EC/DB$	$AD/DB=AE/EC$ Match upper-to-lower on both sides.
05 · Data A In the area proof, $[ADE]=9, [BDE]=18$. Find AD/DB.	0.5 Shared altitude gives base ratio equal to area ratio.
06 · Data A $DE \parallel BC$, $AD/AB=2/5$ and $AC=15$. Find EC.	9 In triangle ABC with D on AB and E on AC, similarity gives $AE = (2/5) \times 15 = 6$. The remaining length is $EC = 15 - 6 = 9$.



This rectangle is a twofold enlargement of a 3 by 2 rectangle. How do the areas compare?

Twenty-four squares versus six. Doubling both directions multiplies area by four.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Data A

point arrangement

Point	D	E
Location	Between A and B	Between A and C

In triangle ABC, D is an internal point of AB and E is an internal point of AC. Use only the parallelism or ratios explicitly given in the question.

Try the problem	Solution and reasoning
07 May you set AD/DB equal to AE/AC for a parallel cut?	Those fractions compare different parts. Match part to remainder on both sides, or part to whole on both.
08 A median divides a triangle into two parts. Why are the areas equal?	The two bases on the same line have equal length, and both triangles share the same perpendicular height.
09 · Data A In ΔABC, $D \in AB, E \in AC, DE \parallel BC$. $AD=5, DB=10, AE=6$. Find EC.	12 $AD/DB = 1/2 = 6/EC$.
10 · Data A $DE \parallel BC$ with $D \in AB, E \in AC$. $AD=3, AB=9, AC=18$. Find AE.	6 The whole-side fraction is 1/3.
11 · Data A In ΔABC, $D \in AB, E \in AC, DE \parallel BC$. $AD=7, DB=14, AE=6$. Find EC.	12 $AD/DB = 1/2 = 6/EC$.

Area ratio = 25:9
↓
Length ratio = 5:3

Why take a square root?

A two-dimensional area uses the length scale twice. Lengths are positive.

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