MATHS FIELD GUIDE · GRADE 10

Arithmetic sequences and the nth term

OPENING PROBLEM

What do the four gaps show?

7 11 15 19 23

02 Distinguish a term from a total
03 Rules and worked cases
04 From the givens to the answer
05 Pair the ends to find a sum
06 Check your reasoning
07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Distinguish a term from a total

In an arithmetic progression, $a_n = a_1 + (n - 1)d$.

1

7 11 15 19 23

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

2

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

3

$7 + 4(n - 1) = 39$

↓

$4(n - 1) = 32$

↓

$n = 9$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

4

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

$d = 0$
a constant AP.

$d < 0$
each term is smaller than its predecessor.

CHAPTER PROBLEM

An AP starts 4, 7, 10. Find its 10th term.

31. The common difference is 3. The tenth term is nine jumps from the first, so it is $4 + 9 \times 3 = 31$.

MATHS REFERENCE · GRADE 10

Arithmetic sequences and the nth term

Rules, conditions and worked cases

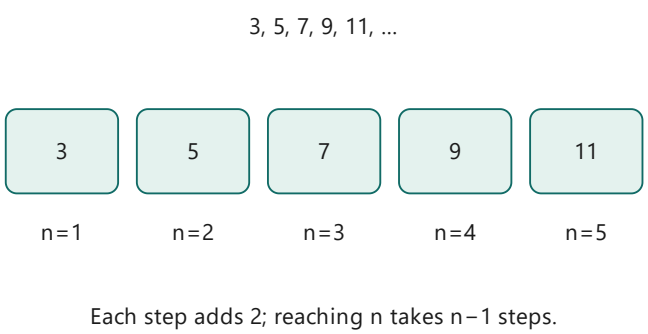
Recognise the case	Mathematical reasoning
Forward calculation	a = 12 and d = −3 give term 7 = 12 + 6(−3) = −6.
Recover d	Terms 3 and 8 are 11 and 31. Five equal jumps span 20, so d = 4.
Membership	For 2, 7, 12, ..., solve 2 + 5(n − 1) = 43. n is not a whole number, so 43 is absent.

Read the representations

An AP changes by a fixed difference

An arithmetic progression is a sequence in which each consecutive difference is the same. Difference can be positive, negative or zero.

Starting at a, reaching term n requires n−1 equal steps: $a_n = a + (n-1)d$.



Position is not the value

For 4,7,10,13,... the first term is 4 and common difference 3.

The fifth term is 16. The value 5 is not the fifth term, and a_5 does not mean 5a.

3 7 11 15 ?

What is the next term if the rule is add four?

Nineteen. Each gap is four.

WORKED EXAMPLES

From the givens to the answer

Find the 18th term of 7,3,−1,...

1. a=7 and d=−4. There are 17 steps to term 18.
2. $a_{18}=7+17(-4)=-61$.
3. Check the rule at n=1 and n=2: it gives 7 and 3.

Find a term from two clues

The 4th term is 13 and the 9th is 28. What is the 20th?

1. Five equal jumps add 15, so each jump adds 3. From term 9 to term 20 there are eleven jumps. The 20th term is 28 + 33 = 61.

Check another case

Example 1

For $a_n=7n+8$, find the common difference.

7

Increasing n by 1 increases the value by d.

Example 2

An AP has a=30 and d=−8. Find a_6.

-10

There are five negative steps.

A term lies halfway between two others

Term 3 is 11 and term 11 is 43. Eight jumps add 32, so each jump adds four. Term 7 is four jumps from term 3 and equals 27.

The terms on either side of term 7 have matching gaps. The average of 11 and 43 is also 27.

Position	1	2	3	4
Term	3	7	11	15

How do you reach term 4 straight from term 1?

Start at 3 and add three jumps of 4. The result is 15.

ILLUSTRATED PROBLEM SHEET

Pair the ends to find a sum

Forward and reverse copies of an AP have equal column sums. Divide their combined total by two.

1

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

Find the sum of the forward row.

Four pairs give 52, twice the sum. The sum is 26.

2

$$1 + 2 + \cdots + 20$$
$$\downarrow$$
$$20 \times 21 \div 2$$
$$\downarrow$$
$$210$$

Which two terms form the first pair?

1 and 20. Every pair of opposite positions totals 21.

3

$$4 + 7 + 10 + 13 + 16$$
$$\downarrow$$
$$2 \times 20 + 10$$
$$\downarrow$$
$$50$$

What if one middle term is left over?

Add the two outer pairs and then the middle 10. The average is 10.

4

$$S_{10} = 190$$
$$\downarrow$$
$$S_9 = 153$$
$$\downarrow$$
$$a_{10} = 37$$

How can two totals reveal just one term?

The first nine terms occur in both totals. Subtraction leaves the tenth alone.

Five equally spaced terms
total = five times the middle term.

Six equally spaced terms
total = three times the sum of the endpoints.

CHAPTER PROBLEM

For $a_n=4n+5$, find the common difference.

4. Increasing n by 1 increases the value by d.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is 40 a term of this progression?	No. Its difference from 7 is 33, which is not a whole number of four-unit gaps.
02 The first n natural numbers sum to 55. Find n.	$10 \times 11 \div 2 = 55$. The sums strictly increase for positive n, so n = 10.
03 An AP has a=30 and d=−6. Find a_6.	0 There are five negative steps.
04 Which sequence is an AP? A) 2,4,8,16 B) 3,6,9,12 C) 1,4,9,16 D) 1,1/2,1/3,1/4	3,6,9,12 Subtract neighbours in 3, 6, 9, 12. Each difference is three, so one fixed addition generates the sequence.
05 For $a_n=8n+9$, find a_1.	17 Substitute n=1; the constant is not the first term.
06 In an AP with d=2, find $a_{12}-a_5$.	14 Seven equal steps separate the positions.

Which terms are negative?

A sequence begins 26, 22, 18. Its nth term is 26 − 4(n − 1). It first turns negative when n − 1 exceeds 6.5.

Term 7 is 2 and term 8 is −2. Checking the neighbours catches an off-by-one error.

22 17 12 7 ?

Can the jump be negative?

Yes. Subtract five each time. The next term is 2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Derive $a_n=a+(n-1)d$ without memorising it.	Term 1 is the starting value. Each move adds d; there are n−1 moves from position 1 to n.
08 Is 8,8,8,8,... an AP? Explain.	All consecutive differences equal 0. It is an AP with first term 8 and d=0.
09 An AP starts 5, 9, 13. Find its 10th term. 5 9 13 ...	41 The first term is 5 and the common difference is 4. Nine jumps bring you to position ten: 5 + 9 × 4 = 41.
10 For $a_n=5n+6$, find the common difference.	5 Increasing n by 1 increases the value by d.
11 An AP starts 7, 13, 19. Find its 10th term. 7 13 19 ...	61 Each jump adds six. From position one to position ten there are nine jumps, giving 7 + 9 × 6 = 61.

Input	1	2	3
Output	3	5	7

Which rule doubles then adds one?

Output = 2 × input + 1. Input 6 gives 13.

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