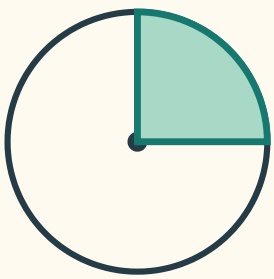
MATHS FIELD GUIDE · GRADE 10

# Arc length and sector area

OPENING PROBLEM

## For radius 8 cm, find the quarter arc.



90°

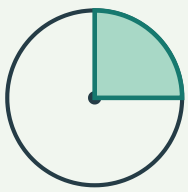
02 Match the fraction of a turn
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07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

# Match the fraction of a turn

For a sector of angle θ in degrees, multiply either the full circumference or full area by θ/360.

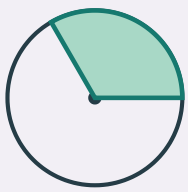
1




**For radius 8 cm, find the quarter arc.**

A quarter of 16π cm is 4π cm.

2




**For radius 6 cm, find the sector area.**

120/360 is one third. A third of 36π is 12π cm².

3




**What if the major sector is shaded?**

Use the full 270/360 = 3/4. The smaller 90° angle refers to the unshaded part.

4




**Find the minor segment area for radius 4 cm.**

Quarter disk 4π minus right triangle area 8 gives (4π − 8) cm².

**60° sector**
**one sixth of the disk.**

**300° sector**
**five sixths of the disk.**

CHAPTER PROBLEM

**A sector angle is 72°. What fraction of the circle does it occupy?**

1/5. 72/360 = 1/5.

MATHS REFERENCE · GRADE 10

# Arc length and sector area

## Rules, conditions and worked cases

| Recognise the case | Mathematical reasoning |
|---|---|
| **Arc length** | A 72° arc of radius 10 is one fifth of 20π, giving 4π length units. |
| **Sector area** | The same fifth of 100π gives 20π square units. |
| **Sector perimeter** | Include both radii with the arc: 10 + 10 + 4π. Arc length alone traces only the curved edge. |

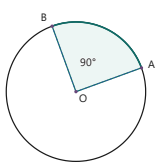
## Read the representations

### A sector takes a fraction of one complete turn

A sector of central angle θ° occupies fraction θ/360 of its circle. Equal central-angle parts give equal arcs and equal sector areas.

Multiply the full circumference 2πr by that fraction for arc length. Multiply the full area πr² by the same fraction for sector area.

Arc length = (θ/360)2πr; sector area = (θ/360)πr². The angle is measured at the centre, in degrees.



Boundary: one arc + two radii

### The two radii belong to sector perimeter

The curved arc is only part of the boundary. A sector with angle below 360° also has two straight radial edges.

Its perimeter is arc length + 2r. For a semicircle this becomes half the circumference plus the diameter.

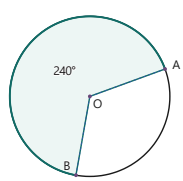
At a full 360° turn the sector is the whole disk. The radial cut disappears, so its boundary is just the circumference.

### A major sector uses the larger angle

The two sectors with the same radius boundaries complete the disk. Their angles sum to 360° and their areas sum to πr².

Use the reflex angle for the major sector. Replacing it by the smaller angle calculates the other region.

The two arc lengths sum to the circumference. Each separate sector also has its own two radial boundary edges.



Boundary: one arc + two radii



r = 3, h = 5

**Find the dimensions of its unrolled wall.**

Height 5 and width 6π. The curved area is 30π square units.

WORKED EXAMPLES

# From the givens to the answer

### Find both measures without mixing their units

1. A sector has radius 6 cm and angle 120°. Its fraction is 120/360 = 1/3.
2. The arc length is (1/3) × 12π = 4π cm. The sector area is (1/3) × 36π = 12π cm².
3. The same fraction is used, but the complete quantities and units are different.

### A turn fraction can be given without degrees

1. A sprinkler rotates through 3/8 of a complete turn with reach 8 m. The model assumes a circular sweep and uniform reach.
2. Its angle is (3/8) × 360° = 135°. The arc length is (3/8) × 16π = 6π m.
3. The swept sector area is (3/8) × 64π = 24π m². Water coverage is an area, while a border around the sweep is a perimeter.

### Is a sector's perimeter the same as its arc length?

A learner reports only the curved length when asked for a sector's full boundary.

1. Trace the boundary from one rim endpoint to the centre, out to the other endpoint, and back along the arc.
2. For a proper sector, the two straight pieces each have length r.
3. Add 2r to the arc length; do not add the whole circumference.

## Check another case

### Example 1

A 45° sector has radius 16 cm and arc length kπ cm. Find k.

4

One eighth of 32π is 4π.

### Example 2

A 210° sector has radius 12 cm and area kπ cm². Find k.

84

(7/12) × 144π = 84π.




### A cone has base radius 3 and slant height 9. Why is the sector angle 120°?

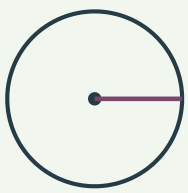
A full radius-9 circle has circumference 18π. The rim needs 6π, or one third of a turn.

ILLUSTRATED PROBLEM SHEET

# Read a circle from its radius

Circumference is the distance around. Area measures the region inside.

1

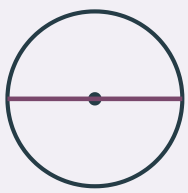


r = 4 cm

### Find both circumference and area.

C = 8π cm and A = 16π cm². Notice the different units.

2



d = 10 cm

### What error follows from treating diameter as radius?

Actual area is 25π cm². Using 10 as radius gives 100π, four times too large.

3

| Radius | r | 2r |
|---|---|---|
| Boundary | 2πr | 4πr |
| Area | πr² | 4πr² |

### Which quantity doubles, and which quadruples?

Boundary scales with one length. Area scales in two directions.

4

Outer radius 5

↓

Inner radius 3

↓

Area = (25 − 9)π

### How much area does a circular ring cover?

16π square units. Subtract the disks' areas, not their radii before squaring.

**Radius 1**
**circumference 2π, area π.**

**Radius 2**
**circumference 4π, area 4π.**

CHAPTER PROBLEM

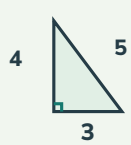
### A 90° sector has radius 8 cm. Its arc length is kπ cm. Find k.

4. One quarter of 16π is 4π.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **01**<br>Does a sector's perimeter equal its arc length? | Include its two straight radii for a non-full sector. Arc length alone counts only the curved edge. |
| **02**<br>A wheel has diameter 0.7 m and rolls ten turns without slipping. How far does it go? | Ten circumferences give 7π metres, close to 22 metres. |
| **03**<br>A 60° sector has radius 12 cm and area kπ cm². Find k.<br> | 24<br>One sixth of 144π is 24π. |
| **04**<br>A 150° sector has radius 6 cm and arc length kπ cm. Find k.<br> | 5<br>(150/360) × 12π = 5π. |
| **05**<br>A 90° sector has radius 10 cm. Which is its exact perimeter?<br>A) 5π cm  B) 25π cm  C) 20π + 20 cm<br>D) 5π + 20 cm | 5π + 20 cm<br>Arc = (1/4)20π = 5π; add the two radii, totalling 20. |
| **06**<br>A circular sweep covers 7/12 of a full turn. Find its central angle in degrees. | 210°<br>(7/12) × 360 = 210°. |




**A cone has radius 3 and vertical height 4. Which measure is the sector radius?**

The slant height 5. It runs along the surface from tip to rim.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

# Check your reasoning

| Try the problem | Solution and reasoning |
|---|---|
| **07**<br>A semicircular region has radius 7 cm. Using π ≈ 22/7, estimate its perimeter in cm. | 36<br>The semicircular arc is about 22 cm; add diameter 14 cm to get about 36 cm. |
| **08**<br>Why is the turn fraction the same for arc length and sector area? | Equal angular divisions divide the circumference into equal arcs.<br>The same rotations divide the disk into equal-area sectors.<br>A chosen fraction of the divisions therefore gives the same fraction of each complete measure. |
| **09**<br>Why does the whole disk's perimeter not equal circumference plus two radii? | A whole disk has only its circular outside boundary.<br>At a full turn the radial edges coincide internally and are not exposed boundary edges.<br>The 2r addition applies to a proper sector with two distinct radial edges, not to the full disk. |
| **10**<br>Explain how to find a proper sector's full perimeter from its radius and central angle. | Use the angle as a fraction of 360°.<br>Multiply that fraction by 2πr to find the arc length.<br>Add two radius lengths because both straight radial edges belong to the boundary. |
| **11**<br>A 240° sector has radius 9 cm and area kπ cm². Find k.<br> | 54<br>Two-thirds of 81π is 54π. |

**Curved area = π × 3 × 5**
↓
**Base area = 9π**
↓
**Total = 24π**

### What changes if the cone is open at the base?

Only the 15π curved surface remains.

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