MATHS FIELD GUIDE · GRADE 10

AP sums and inverse sum problems

OPENING PROBLEM

Can the sum of the first n positive even numbers be 25 for any whole number n?

2 4 6 8 10

02 Work back from an arithmetic total
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07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Work back from an arithmetic total

Use $S_n = n(a_1 + a_n)/2$. A valid term count is a positive integer.

1

Rows 4, 8, 12, ...

↓

$S_n = 2n(n + 1)$

↓

Total 84

How many rows were used?

$n(n + 1) = 42$. Six times seven gives 42, hence six rows. Check 4 + 8 + 12 + 16 + 20 + 24 = 84.

2

$S_7 = 91$

↓

$S_6 = 69$

↓

$a_7 = 22$

Why can subtraction recover the next term?

The first six appear in both totals. Only the seventh remains.

3

10 terms

↓

First 3, total 210

↓

3 + last = 42

Find the last term.

The mean term is 21. The two endpoints sum to 42, so the last is 39. Nine gaps give d = 4.

4

2 4 6 8 10

Can the sum of the first n positive even numbers be 25 for any whole number n?

The first four sum to 20. The first five sum to 30. All terms are positive, so 25 is skipped.

$S_5 = 40$ and $S_4 = 29$
term 5 is 11.

$a_5 = 11$ and $S_4 = 29$
$S_5 = 40$.

CHAPTER PROBLEM

A student saves 10,15,20,... per week. Find the total after 6 weeks.

135. The sixth amount is 35. The six amounts form three pairs of 45, giving a total of 135.

MATHS REFERENCE · GRADE 10

AP sums and inverse sum problems

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Forward sum	First 12 terms of 3,7,11,... end at 47. Sum = 12(3 + 47)/2 = 300.
Unknown term count	For 2 + 4 + ... + 2n = 72, solve n(n + 1) = 72. Only n = 8 is a positive count.
Recover one term	$S_n - S_{n-1}$ equals the nth term. If $S_n = n^2$, the nth term is 2n − 1.

Read the representations

A cumulative quantity calls for a sum

If a student saves 20 in week 1 and 5 more each week, the weekly amounts form an AP.

The total over n weeks is S_n, whereas the amount in week n is a_n. State the units and starting position.

Different questions, different formulae

Question	Model
How much in week n?	20+5(n−1)
How much by the end of week n?	n[40+5(n−1)]/2

An inverse total needs a feasible count

Equating S_n to a target often gives a quadratic equation.

Reject negative counts and check whether the positive root is an integer. If a threshold is requested, compare neighbouring integer totals.

7	11	15	19	23

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

WORKED EXAMPLES

From the givens to the answer

Rows of seats: 12,15,18,...; total 117

1. Here a=12,d=3. The sum equation is n[24+3(n−1)]/2=117.
2. Divide by 3 and simplify: $n^2+7n-78=0=(n+13)(n-6)$.
3. The feasible count is n=6. Check: last row 27 and total 6(12+27)/2=117.

How many rows did the hall use?

Rows contain 6, 10, 14, ... seats. The hall has 70 seats. Find the row count.

1. With n rows the total is n(2n + 4). Solve $2n^2 + 4n - 70 = 0$, giving (n − 5)(n + 7) = 0. Only five rows are possible. Check the last row has 22 seats, and 5 × 14 = 70.

Check another case

Example 1

Rows have 8,10,12,... seats. How many seats in row 10?

26

The question asks for a term.

Example 2

Rows have 8,10,12,... seats. Find the total in the first 11 rows.

198

S=n[16+2(n−1)]/2.

Recover a term from two totals

The first 12 terms add to 354 and the first 11 add to 308. Subtracting leaves only term 12, which is 46.

If the common difference is 4, step back eleven times to find the first term 2. The sum of twelve terms is 12(2 + 46)/2 = 288, so those three clues cannot all describe one AP.

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

ILLUSTRATED PROBLEM SHEET

Pair the ends to find a sum

Forward and reverse copies of an AP have equal column sums. Divide their combined total by two.

1

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

Find the sum of the forward row.

Four pairs give 52, twice the sum. The sum is 26.

2

$$1 + 2 + \cdots + 20$$
$$\downarrow$$
$$20 \times 21 \div 2$$
$$\downarrow$$
$$210$$

Which two terms form the first pair?

1 and 20. Every pair of opposite positions totals 21.

3

$$4 + 7 + 10 + 13 + 16$$
$$\downarrow$$
$$2 \times 20 + 10$$
$$\downarrow$$
$$50$$

What if one middle term is left over?

Add the two outer pairs and then the middle 10. The average is 10.

4

$$S_{10} = 190$$
$$\downarrow$$
$$S_9 = 153$$
$$\downarrow$$
$$a_{10} = 37$$

How can two totals reveal just one term?

The first nine terms occur in both totals. Subtraction leaves the tenth alone.

Five equally spaced terms
total = five times the middle term.

Six equally spaced terms
total = three times the sum of the endpoints.

CHAPTER PROBLEM

Rows have 8,10,12,... seats. How many seats in row 7?

20. The question asks for a term.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Why reject a negative solution for n even if it satisfies a quadratic for the sum?	A negative count does not identify any initial group of terms in the sequence.
02 The first n natural numbers sum to 55. Find n.	$10 \times 11 \div 2 = 55$. The sums strictly increase for positive n, so $n = 10$.
03 Rows have 8,10,12,... seats. Find the total in the first 9 rows. 8 10 12 ...	144 $S=n[16+2(n-1)]/2$.
04 A total of 110 blocks is arranged in rows 2,4,6,... . How many complete rows?	10 $S=n(n+1)$; check the positive integer solution.
05 An inverse sum equation gives $n=-8$ or $n=5$. Which is feasible for the number of rows? A) −8 B) 5 C) Both D) Neither	5 A count of rows is a positive integer.
06 An object travels 3 m in its first second and 2 m more in each successive second. Find distance travelled during second 5.	11 Use the term $3+(n-1)2$, not total distance.

$$7 + 4(n - 1) = 39$$
$$\downarrow$$
$$4(n - 1) = 32$$
$$\downarrow$$
$$n = 9$$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Explain how to find the first week when accumulated savings exceed a target.	Use S_n for the cumulative amount. Solve or estimate the boundary then compare neighbouring integer week totals.
08 Can a non-integer solution for n be rounded when an exact total is specified?	No: rounding changes the sum and may violate the exact condition. State that no complete integer number of stages meets the target unless a check proves otherwise.
09 A student saves 10,15,20,... per week. Find the total after 7 weeks.	175 Use the AP sum with a=10,d=5.
10 Rows have 8,10,12,... seats. How many seats in row 8? 8 10 12 ...	22 The eighth row is seven increases after the first. It has 8 + 7 × 2 = 22 seats.
11 A student saves 10,15,20,... per week. Find the total after 9 weeks.	270 The ninth week contributes 10 + 8 × 5 = 50. The mean of the first and last contributions is 30, so the total is 9 × 30 = 270.

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

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