MATHS FIELD GUIDE · GRADE 10

AP modelling and linked representations

OPENING PROBLEM

What do the four gaps show?

7 11 15 19 23

02 Distinguish a term from a total
03 Rules and worked cases
04 From the givens to the answer
05 Pair the ends to find a sum
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07 Check your reasoning

ILLUSTRATED PROBLEM SHEET

Distinguish a term from a total

In an arithmetic progression, $a_n = a_1 + (n - 1)d$.

1

7 11 15 19 23

What do the four gaps show?

The common difference is 4. Reaching term 5 takes four jumps.

2

$a_1 = 7$ and $d = 4$

↓

$a_{12} = 7 + 11 \times 4$

↓

51

Why use eleven instead of twelve?

The first term is already present before making any jumps.

3

$7 + 4(n - 1) = 39$

↓

$4(n - 1) = 32$

↓

$n = 9$

Does 39 occur?

Yes. The position is a positive integer. The adjacent terms are 35 and 43.

4

Position	3	8
Term	15	35

Recover the difference without knowing term 1.

Five gaps add 20, so d = 4. Step back twice from 15 to get 7.

d = 0
a constant AP.

d < 0
each term is smaller than its predecessor.

CHAPTER PROBLEM

A pattern has 4,7,10,... sticks. Find sticks at stage 5.

16. a=4,d=3 gives 4+3(n−1).

MATHS REFERENCE · GRADE 10

AP modelling and linked representations

Rules, conditions and worked cases

Recognise the case	Mathematical reasoning
Costs by row	18 seats in row one and two more per row give $a_n = 18 + 2(n - 1)$. Row ten has 36.
Total seats	Ten rows average $(18 + 36)/2 = 27$ seats. Total 270.
Inspect a graph	Plot row number against seats at integer inputs. The slope is two seats per row; a fractional row has no physical meaning.

Read the representations

A model begins with the changing quantity

Constant additions give an AP for the stage values. Constant multiplication does not.

A fixed initial fee plus equal charges gives a linear total; accumulating AP stage values gives a quadratic total.

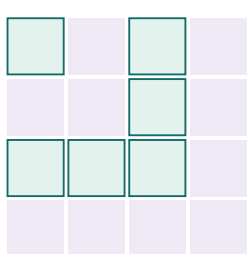
Compare the change

Situation	Sequence type
Add 4 each stage	AP
Double each stage	Not an AP (unless the start is 0)
Sum the first n odd counts	Quadratic total n^2

Document an investigation with a general argument

Draw successive rows of 1,3,5,... dots around a square. The first n odd counts make n^2 dots.

A picture suggests the rule; $S_n=n[2+(n-1)2]/2=n^2$ proves it for positive integer n.



$1 + 3 + 5 + 7 = 4^2 = 16$

Each new border adds the next odd number.

3 7 11 15 ?

What is the next term if the rule is add four?

Nineteen. Each gap is four.

WORKED EXAMPLES

From the givens to the answer

Tiles around a growing row of squares

1. A strip of n adjacent unit squares needs 3n+1 unit matchsticks: n top, n bottom and n+1 vertical.
2. Thus the counts 4,7,10,... form an AP with a=4,d=3.
3. For 12 squares, 3(12)+1=37. Count shared edges only once.

Read a savings graph

At the end of week 1 Mira has ₹80. She adds ₹15 each week. Is the graph a continuous line?

1. Week n has balance 80 + 15(n − 1). At week 5 it is ₹140. Plot dots at whole week numbers. The model supplies no end-of-week balance at week 2.5.

Check another case

Example 1

Rows 1,3,5,... form a square. Find the total dots in 9 rows.

81

The sum of the first n positive odd integers is n^2.

Example 2

A taxi charges 30 initially plus 12 per kilometre. Find the charge for 4 kilometres.

78

The fixed charge is not repeated for each kilometre.

Match a pattern to the right quantity

An auditorium row starts at 12 seats and each new row gains three. Row 6 holds 12 + 5 × 3 = 27 seats.

Across the first six rows there are 6(12 + 27)/2 = 117 seats. Decide whether the question asks for one row or all rows so far.

Position	1	2	3	4
Term	3	7	11	15

How do you reach term 4 straight from term 1?

Start at 3 and add three jumps of 4. The result is 15.

ILLUSTRATED PROBLEM SHEET

Pair the ends to find a sum

Forward and reverse copies of an AP have equal column sums. Divide their combined total by two.

1

Forward	2	5	8	11
Reverse	11	8	5	2
Pair	13	13	13	13

Find the sum of the forward row.

Four pairs give 52, twice the sum. The sum is 26.

2

$$1 + 2 + \cdots + 20$$
$$\downarrow$$
$$20 \times 21 \div 2$$
$$\downarrow$$
$$210$$

Which two terms form the first pair?

1 and 20. Every pair of opposite positions totals 21.

3

$$4 + 7 + 10 + 13 + 16$$
$$\downarrow$$
$$2 \times 20 + 10$$
$$\downarrow$$
$$50$$

What if one middle term is left over?

Add the two outer pairs and then the middle 10. The average is 10.

4

$$S_{10} = 190$$
$$\downarrow$$
$$S_9 = 153$$
$$\downarrow$$
$$a_{10} = 37$$

How can two totals reveal just one term?

The first nine terms occur in both totals. Subtraction leaves the tenth alone.

Five equally spaced terms
total = five times the middle term.

Six equally spaced terms
total = three times the sum of the endpoints.

CHAPTER PROBLEM

Rows 1,3,5,... form a square. Find the total dots in 6 rows.

36. The sum of the first n positive odd integers is n^2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
01 Is 40 a term of this progression?	No. Its difference from 7 is 33, which is not a whole number of four-unit gaps.
02 The first n natural numbers sum to 55. Find n.	$10 \times 11 \div 2 = 55$. The sums strictly increase for positive n, so n = 10.
03 Which is an AP model? A) A balance doubles each year. B) A ladder has rungs at equal 25 cm spacings. C) Square areas are 1,4,9,16. D) A quantity halves each day.	A ladder has rungs at equal 25 cm spacings. Equal rung spacings give a constant additive difference in height.
04 A taxi charges 30 initially plus 12 per kilometre. Find the charge for 9 kilometres.	138 The fixed charge is not repeated for each kilometre.
05 For stage values $a_n=5n-2$, find $a11-a10$.	5 The stage values have constant first difference 5.
06 $A_n=n^2$ is a cumulative total. Find $A4-A3$.	7 $n^2-(n-1)^2=2n-1$.

22 17 12 7 ?

Can the jump be negative?

Yes. Subtract five each time. The next term is 2.

QUESTION BANK · FOLD OR COVER THE RIGHT COLUMN

Check your reasoning

Try the problem	Solution and reasoning
07 Investigate why the sum of the first n odd integers is n^2.	Draw successive odd borders around a square or organise dot rows. Derive the AP sum algebraically. State n is a positive integer and distinguish the nth odd term from its sum.
08 Explain why 1,4,9,16,... is not an AP even though it has a simple rule.	Its differences are 3,5,7,... rather than constant. Having a formula does not by itself make a sequence arithmetic.
09 A pattern has 4,7,10,... sticks. Find sticks at stage 6. 4 7 10 ...	19 a=4,d=3 gives 4+3(n−1).
10 Rows 1,3,5,... form a square. Find the total dots in 7 rows.	49 The sum of the first n positive odd integers is n^2.
11 A pattern has 4,7,10,... sticks. Find sticks at stage 8. 4 7 10 ...	25 a=4,d=3 gives 4+3(n−1).

Input	1	2	3
Output	3	5	7

Which rule doubles then adds one?

Output = 2 × input + 1. Input 6 gives 13.

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