Turn an observation into a proof

Write your reasoning in the space below. Turn the page when you are ready to check.

Five possible remainders

0 | 1 | 2 | 3 | 4

1. Six tickets, five boxes

Six different integer ticket numbers are drawn. Prove that two of them differ by a multiple of 5.

2. Is six necessary?

Can five different ticket numbers avoid having a pair whose difference is a multiple of 5? Give an example or a proof.

3. Change the divisor

What is the fewest number of distinct integer tickets that guarantees a pair whose difference is divisible by 8? Prove that your number is the fewest.

Worked answers

Turn an observation into a proof

1. Six tickets, five boxes

Group the numbers by the remainder they leave on division by 5.

There are only five possible remainders: 0, 1, 2, 3 and 4. Putting six numbers into five groups puts two in the same group. Write those numbers as 5a + r and 5b + r. Their difference is 5(a - b).

2. Is six necessary?

Choose one number from each remainder group.

Yes. Take 10, 11, 12, 13 and 14. Every positive difference lies between 1 and 4. None is divisible by 5. This shows why the sixth ticket matters.

3. Change the divisor

A minimum needs both a guarantee and an example one below it.

Nine tickets force two into the same one of eight remainder groups, so their difference is divisible by 8. Eight tickets can be 1 through 8, giving only differences 1 through 7. Thus nine is the minimum.