Write both halves of an extremal proof

Write your reasoning in the space below. Turn the page when you are ready to check.

A shelf with twelve slots

1	2	3	4	5	6	7	8	9	10	11	12

1. Design a shelf

A shelf has 12 numbered slots in a row. Choose as many slots as possible with at least two unused slots between any two chosen slots. Prove your maximum.

2. Add one slot

What changes if there are 13 slots with the same rule?

3. Close the shelf into a ring

Place the 13 slots around a circle. Leave at least two unchosen slots between successive chosen slots, including the gap across the join. What is the maximum?

Worked answers

Write both halves of an extremal proof

1. Design a shelf

Split the shelf into consecutive blocks of three.

Each block of three holds at most one chosen slot, so four blocks give an upper bound of four. Slots 1, 4, 7 and 10 achieve four and obey the spacing rule. Both parts are needed.

2. Add one slot

Count the last slot separately.

The first 12 give at most four selections and slot 13 adds at most one. Choose 1, 4, 7, 10 and 13. Five is attainable and no larger number can fit.

3. Close the shelf into a ring

Assign each chosen slot its next two slots clockwise.

Those groups of three are disjoint, so 3k is at most 13 for k chosen slots. Hence k is at most 4. Slots 1, 4, 7 and 10 work: slots 11, 12 and 13 are free before returning to 1.