Heights and distances with trigonometry: Grade 10

Visual learning guide - Maths · Grade 10

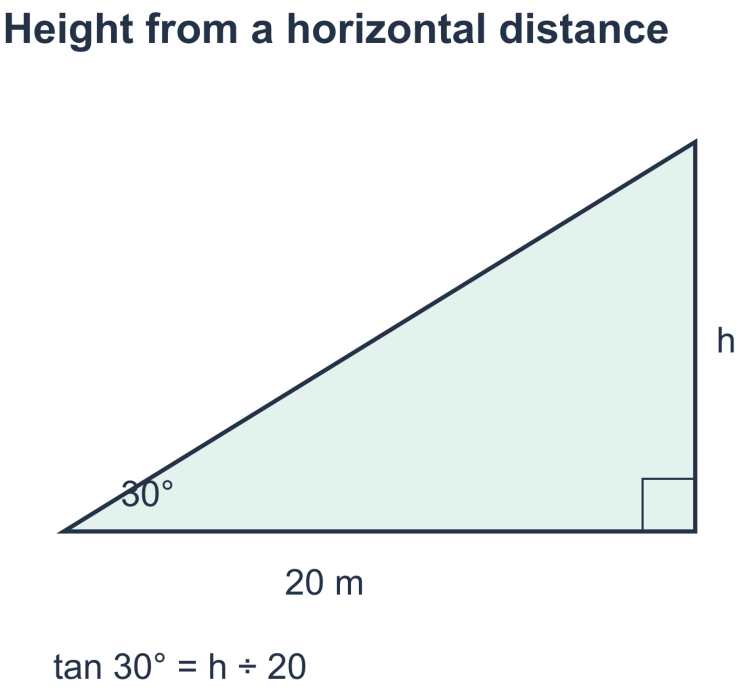
Follow the models and 5 worked cases, then try 14 questions with explanations. Save the guide or worksheet for another day.

Start with the key idea

For a right-triangle height problem, draw the horizontal distance, vertical rise and line of sight. Label the angle at the observer. Choose sine, cosine or tangent according to the known and unknown sides. Add eye height when the vertical rise starts above the ground.

1. Translate the scene into a right triangle

Draw the ground as horizontal and a tower as vertical when the problem states or implies a level-ground, upright-tower model. Their intersection is a right angle. The sloping line from the observer to the top is the line of sight, not the ground distance.



A right triangle has horizontal distance twenty metres, tower height h and a thirty-degree angle at ground level.

The angle of elevation is measured up from the horizontal through the observer's eye. It is not measured from the tower. Labelling the angle at the wrong vertex exchanges opposite and adjacent sides and can invert the calculation.

Write the assumptions beside the sketch: level ground, a vertical object and a straight line of sight. A sloping hillside needs a different model; a neat diagram must not silently replace the actual conditions in the question.

2. Choose a ratio from the sides you have

Relative to the marked angle, the opposite side is the vertical rise, the adjacent side is the horizontal distance and the hypotenuse is the line of sight. Tangent connects opposite and adjacent, so it is often useful when a ground distance is known.

If a ladder length is supplied, that is usually the hypotenuse. Sine connects vertical height to ladder length; cosine connects ground distance to ladder length. A ladder of length ten metres at 30° to horizontal reaches 10 sin 30° = 5 m.

Do not choose a formula merely because the story mentions a tower. Circle the two side quantities involved and select the ratio that contains them. This makes the same method work for trees, ramps, ladders and observation points.

3. Keep exact values until the last step

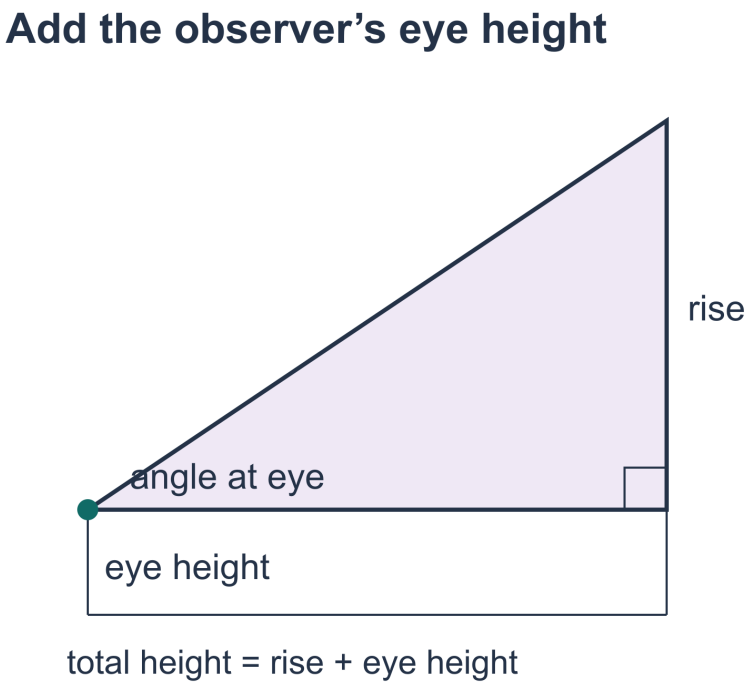
Useful values are tan 30° = 1/√3, tan 45° = 1 and tan 60° = √3. For a rise above a point 20 m away at 30°, h = 20/√3 = 20√3/3 metres. An approximation is about 11.55 m.

Rounding √3 to 1.7 too early changes the final answer unnecessarily. Keep the exact surd through the calculation, then round to the requested precision. If a calculator is used for an angle measured in degrees, its degree mode must match the question.

A quick size check helps. At 45°, rise equals horizontal distance. At an acute angle below 45°, rise is smaller than that distance; above 45°, it is greater. These comparisons can reveal an inverted tangent ratio.

4. Distinguish vertical rise from full object height

Suppose the observer's eye is 1.5 m above level ground. A horizontal line from the eye meets the tower 1.5 m above its base. The triangle's opposite side measures only the rise from that horizontal to the top.



The line of sight begins at eye height; the right triangle measures the rise above that eye level, which must be added to eye height.

At a distance of 10 m and an angle of 45°, the rise is 10 m. The full tower height is therefore 10 + 1.5 = 11.5 m. Returning 10 m would ignore the eye-height information.

For an observer on a raised platform, identify the reference height in the same way. Add the known height above the object's base only when the ground levels and vertical measurements support that addition.

5. Convert an angle of depression carefully

An angle of depression is measured down from a horizontal at the observer. For two parallel horizontal reference lines, the angle of depression equals the corresponding angle of elevation along the same line of sight.

From a 20 m high lookout, an object on level ground is seen at 45° depression. In the right triangle, vertical height and horizontal distance are equal, so the object is 20 m horizontally from the base. The sloping sight distance is different.

Draw both horizontals when the equal-angle relationship is unclear. The equality follows from parallel lines, not from assuming every angle in the picture matches. Keep the right angle at the vertical-ground intersection.

6. Use two observations without inventing a distance

At a near point the elevation is 60°; at a point 20 m farther away on the same straight line it is 30°. Let the nearer horizontal distance be x and the height h, assuming both observations are at ground level.

The equations are $h = x\sqrt{3}$ and $h = (x + 20)/\sqrt{3}$. Equating them gives $3x = x + 20$, hence $x = 10$ and $h = 10\sqrt{3}$ m. Both equations refer to the same height, which is what connects the observations.

Check the geometry before writing x + 20. If the observers stand on opposite sides of the tower, their total separation involves a sum of two distances instead. The words "towards", "away" and "same side" determine the diagram, not a memorised equation.

5 worked examples with reasoning

1. A tree is 15 m away on level ground and its top is at 45° elevation from ground level. Find its height.

1. $\tan 45° = h/15 = 1$.

2. $h = 15$ m.

3. Height equals distance because the right triangle has a 45° acute angle.

2. A 10 m ladder makes 30° with horizontal ground. Find the height reached.

1. The ladder is the hypotenuse.

2. $\sin 30° = h/10$, so $h = 5$ m.

3. Do not use tangent with the ladder length as adjacent.

3. An eye 1.6 m above level ground sees a top at 45° from 12 m away. Find total height.

1. Rise above eye level is $12 \tan 45° = 12$ m.

2. Add the eye height: $12 + 1.6 = 13.6$ m.

3. Check that the observer and object base are on the same ground level.

4. A tower of height 18 m is seen at 60° elevation from ground level. Find horizontal distance.

1. tan 60° = 18/d.

2. d = 18/√3 = 6√3 m.

3. This is about 10.39 m and is smaller than the height.

5. Two ground-level observations on the same side are 20 m apart, with angles 60° nearer and 30° farther. Find tower height.

1. Let near distance x; far distance x + 20.

2. x√3 = (x + 20)/√3 gives x = 10.

3. Height = 10√3 m, about 17.32 m.

Use the ideas without looking at the notes

Draw the scene, mark the observer's horizontal, identify the actual known side and choose a ratio. Check eye height, units and whether the final answer is a vertical or horizontal distance.

Common questions

Do I always use tangent?

No. Choose the ratio from the known side and required side; ladder lengths often involve sine or cosine.

When do I add eye height?

When the triangle measures a rise above eye level and the full object height is required.

Are depression and elevation angles equal?

The corresponding pair is equal when the horizontal reference lines are parallel.

Should I round √3 immediately?

Keep exact values until the final step unless the question specifies an approximation.

Why can two-observer questions have different equations?

Observers on the same side and opposite sides produce different relationships between horizontal distances.

Connect your learning

- Check right-triangle assumptions
- Review angle relationships
- Revisit equation-solving checks

Ask about guided Maths learning if your child needs help explaining these ideas.

About this guide

Original explanations, examples, diagrams and practice by BananaLearn. The grade label indicates the intended learning level; this is not an official textbook extract or exam paper.

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Practice worksheet - Maths · Grade 10

Name: _____ Date: _____

Show your working. Answers and explanations follow on a separate page.

Unless stated otherwise, assume level horizontal ground, vertical objects and observation from ground level. Show a labelled triangle and give units.

Write an answer and your reason before opening the explanation. Check the reasoning as well as the final answer.

1. Which ratio connects opposite and adjacent sides?

2. Where is an elevation angle measured from?

3. From ground level, distance 8 m and elevation 45°. Height?

4. Distance 9 m and elevation 60° from ground level. Height?

5. Height 12 m and elevation 45°. Horizontal distance?

6. Height 10 m and elevation 30°. Horizontal distance?

7. Rise above eye is 7 m and eye height is 1.4 m. Total?

8. A 6 m ladder is at 30° to ground. Height reached?

Practice continued

9. At 60° to ground, what height does the same 6 m ladder reach?

10. A 15 m lookout sees a ground object at 45° depression. Horizontal distance?

11. At 30°, should rise exceed horizontal distance?

12. Can you combine a distance in metres and a height in centimetres directly in a ratio?

13. Does the line of sight represent the horizontal ground distance?

14. Why should level ground be stated?

Answers and explanations

Answer key - Maths · Grade 10

Heights and distances with trigonometry: Grade 10

1. Tangent.

tan θ = opposite/adjacent.

2. The observer's horizontal.

It is measured upward towards the line of sight.

3. 8 m.

tan 45° = 1.

4. 9√3 m.

Height = distance × tan 60°.

5. 12 m.

The two legs are equal.

6. 10√3 m.

10 divided by 1/√3.

7. 8.4 m.

Add the two vertical lengths.

8. 3 m.

6 × sin 30°.

9. 3√3 m.

6 × sin 60°.

10. 15 m.

The matching elevation is 45°.

11. No.

tan 30° is less than one.

12. Convert to matching units first.

A trigonometric ratio compares like units.

13. No.

It is the hypotenuse in this model.

14. It establishes the horizontal-vertical right-triangle model.

Sloping ground changes the geometry.