Quadratic equations by factorisation: Grade 10

Visual learning guide - Maths · Grade 10

Follow the models and 5 worked cases, then try 14 questions with explanations. Save the guide or worksheet for another day.

Start with the key idea

To solve a quadratic by factorisation, write it as $ax^2 + bx + c = 0$ with a nonzero, factor the expression and apply the zero-product property. Set each factor equal to zero, solve the resulting linear equations and check any restrictions from the original problem.

1. Put every term on one side first

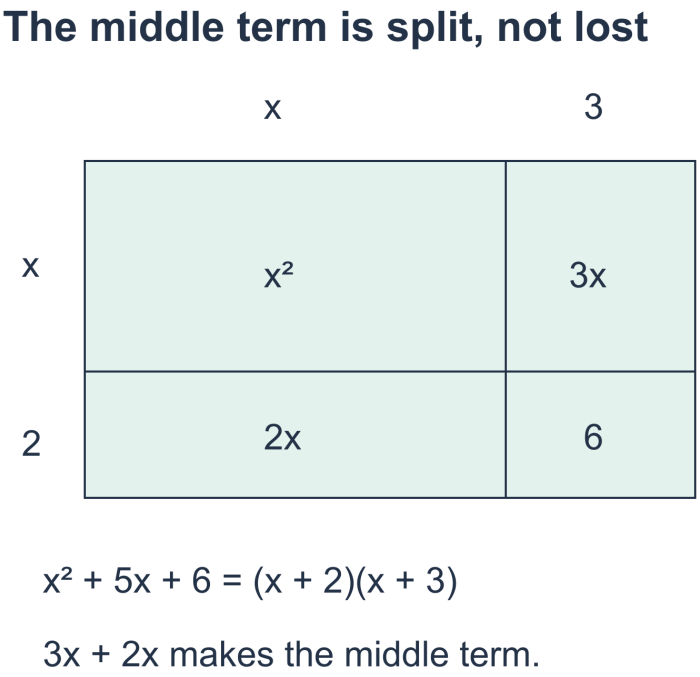
The zero-product property applies to a product equal to zero. For $x^2 = 5x - 6$, subtract $5x$ and add 6 on both sides to obtain $x^2 - 5x + 6 = 0$. Moving a term is shorthand for performing the same operation on both sides.

If the equation is $x(x - 5) = -6$, it is not valid to say $x = -6$ or $x - 5 = -6$. Many products equal -6. Zero is special: a product of real numbers is zero only when at least one factor is zero.

Before factoring, collect like terms and identify a, b and c with their signs. A coefficient of x^2 must remain nonzero. If the squared terms cancel entirely, the resulting equation is linear and should be solved as such.

2. For $x^2 + bx + c$, match a sum and a product

Expanding $(x + p)(x + q)$ gives $x^2 + (p + q)x + pq$. Therefore p and q must add to b and multiply to c. For $x^2 + 5x + 6$, choose 2 and 3: their sum is 5 and their product is 6.



A rectangle splits into x squared, three x, two x and six; its side lengths are x plus two and x plus three.

When c is positive, the two numbers have the same sign. The sign of b tells you which sign. For $x^2 - 5x + 6$, both are negative: -2 and -3. When c is negative, use opposite signs and test which magnitude gives the required sum.

Avoid choosing factors from the constant term alone. For $x^2 + 7x + 12$, both 2 and 6 multiply to 12, but they add to 8. The correct pair is 3 and 4. Expand your proposed brackets before finding roots.

3. Split the middle term when a is not one

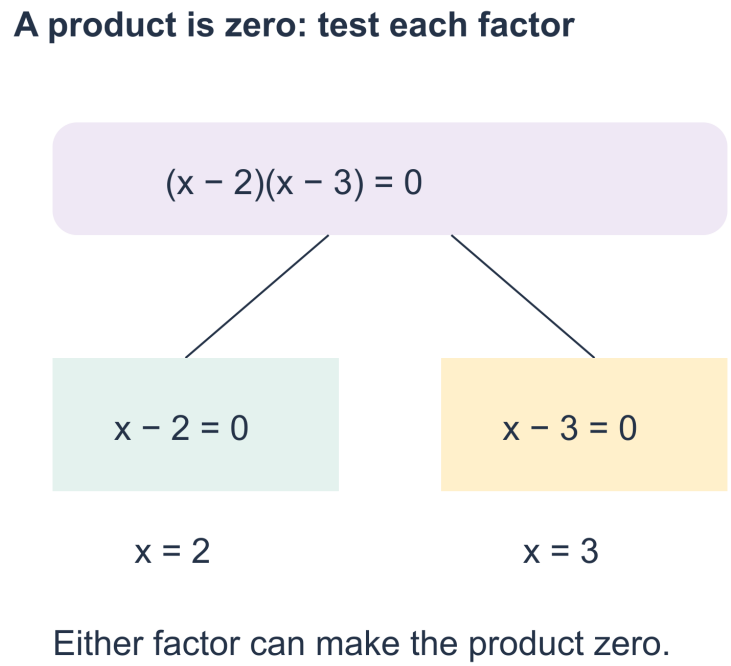
For $2x^2 + 7x + 3$, multiply a and c to get 6. Find two numbers with product 6 and sum 7: 6 and 1. Write $2x^2 + 6x + x + 3$, then group as $2x(x + 3) + 1(x + 3)$.

The shared bracket gives $(2x + 1)(x + 3)$. Splitting $7x$ into $6x + x$ preserves the expression. It does not create extra terms or change the equation; it reveals a common factor in two groups.

If the second group has a negative sign, factor that sign carefully. For $2x^2 - 5x - 3$, split $-5x$ into $-6x + x$: $2x(x - 3) + 1(x - 3)$. Both brackets must match before combining.

4. Use the zero-product property without losing a root

From $(x - 2)(x - 3) = 0$, conclude $x - 2 = 0$ or $x - 3 = 0$. This gives $x = 2$ or $x = 3$. You do not require both brackets to be zero at the same time.



The equation x minus two times x minus three equals zero branches into x equals two or x equals three.

For $x(x - 4) = 0$, the roots are 0 and 4. Dividing both sides by x would silently assume x is nonzero and lose the root 0. Factoring out a common variable is safe; cancelling it without checking zero is not.

A repeated factor such as $(x - 3)^2 = 0$ has one distinct root, 3, repeated twice algebraically. Two factors do not always give two different numbers. Substitute into the original equation to verify the result.

5. Translate a word problem and reject impossible values

A rectangle has width x cm and length $x + 3$ cm. Its area is 40 cm², so $x(x + 3) = 40$. Rearranging gives $x^2 + 3x - 40 = 0$, which factors as $(x + 8)(x - 5) = 0$.

The algebraic roots are -8 and 5. A rectangle cannot have a negative physical width, so choose $x = 5$ cm and length 8 cm. Check both the area, $5 \times 8 = 40$ cm², and the difference of the sides, 3 cm.

State the restriction before selecting the answer. In a different context a negative root may be valid, for example an abstract number problem. Do not discard every negative answer automatically merely because another story involved a length.

6. Recognise when this method is inconvenient

Some quadratics factor neatly with integers; others do not. For $x^2 - 2 = 0$, the real roots are $\sqrt{2}$ and $-\sqrt{2}$, so integer factor-pair searching will not succeed. A square-root method is more direct.

If a factor search fails, recheck signs and coefficients, then choose another appropriate method such as completing the square or the quadratic formula. A failed integer search does not prove that the equation has no real solutions.

Finally substitute each candidate into the equation as it was originally written. This checks algebraic manipulation and helps catch a copied sign. For an equation derived from a story, also state the quantity and unit that the chosen root represents.

5 worked examples with reasoning

1. Solve $x^2 - 7x + 12 = 0$.

1. Find numbers with sum -7 and product 12: -3 and -4.

2. $(x - 3)(x - 4) = 0$, so $x = 3$ or 4.

3. Check: $9 - 21 + 12 = 0$ and $16 - 28 + 12 = 0$.

2. Solve $2x^2 + 7x + 3 = 0$.

1. Split the middle term: $2x^2 + 6x + x + 3$.

2. Factor to $(2x + 1)(x + 3) = 0$.

3. Roots are $-1/2$ and -3; both make the original expression zero.

3. Solve $x^2 = 5x$.

1. Rearrange to $x^2 - 5x = 0$.

2. $x(x - 5) = 0$.

3. Roots are 0 and 5. Do not divide by x and lose zero.

4. A rectangle is 4 cm longer than its width and has area 45 cm^2. Find its dimensions.

1. Let width be $x > 0$. Then $x(x + 4) = 45$.

2. $x^2 + 4x - 45 = (x + 9)(x - 5) = 0$.

3. Choose width 5 cm and length 9 cm. Reject -9 cm; check $5 \times 9 = 45$.

5. Solve $x^2 - 6x + 9 = 0$.

1. Recognise $(x - 3)^2 = 0$.

2. The only distinct root is $x = 3$.

3. Check $9 - 18 + 9 = 0$; it is a repeated root.

Use the ideas without looking at the notes

Rearrange to zero, factor, set each factor to zero and check every root. Keep the algebraic roots separate from the values permitted by the story.

Common questions

Must the right side be zero?

Yes, before applying the zero-product property. Rearrange the equation first.

What if one root is zero?

Keep it unless the original equation or context excludes zero.

Can both roots be negative?

Yes. For example, $x^2 + 5x + 6 = 0$ has roots -2 and -3.

Does every quadratic factor into integer brackets?

No. Some need irrational or non-real roots, or another solving method.

Why check the original equation?

It catches sign errors and confirms that a candidate satisfies the actual question.

Connect your learning

- Expand brackets and recognise identities
- Compare linear equations
- Use the quadratic formula when needed

Ask about guided Maths learning if your child needs help explaining these ideas.

About this guide

Original explanations, examples, diagrams and practice by BananaLearn. The grade label indicates the intended learning level; this is not an official textbook extract or exam paper.

Quadratic equations by factorisation: Grade 10

Practice worksheet - Maths · Grade 10

Name: _____ Date: _____

Show your working. Answers and explanations follow on a separate page.

Try each task before opening its answer. Give an example or reason, not only a label.

Write an answer and your reason before opening the explanation. Check the reasoning as well as the final answer.

1. Solve $x^2 + 5x + 6 = 0$.

2. Solve $x^2 - x - 6 = 0$.

3. Solve $x^2 - 9 = 0$.

4. Solve $x^2 + 4x = 0$.

5. Solve $2x^2 - 5x - 3 = 0$.

6. Solve $x^2 + 6x + 9 = 0$.

7. Write $x^2 + 2 = 3x$ in zero form.

8. Solve $x^2 - 3x + 2 = 0$.

Practice continued

9. Are 2 and 6 the pair for $x^2 + 7x + 12$?

10. Why can dividing $x(x - 7) = 0$ by x be unsafe?

11. A square has area 49 cm^2. What is its side length?

12. A rectangle has width x and length $x + 1$, area 20 square units. Find its dimensions.

13. Does no integer factor pair mean no real roots?

14. Can $ab = 6$ justify $a = 0$ or $b = 0$?

Answers and explanations

Answer key - Maths · Grade 10

Quadratic equations by factorisation: Grade 10

1. -2 and -3.

$(x + 2)(x + 3) = 0$.

2. 3 and -2.

$(x - 3)(x + 2) = 0$.

3. 3 and -3.

Use $(x - 3)(x + 3)$.

4. 0 and -4.

Factor $x(x + 4)$.

5. 3 and $-1/2$.

$(2x + 1)(x - 3) = 0$.

6. -3, a repeated root.

The expression is $(x + 3)^2$.

7. $x^2 - 3x + 2 = 0$.

Subtract $3x$ from both sides.

8. 1 and 2.

$(x - 1)(x - 2) = 0$.

9. No.

Their sum is 8. Use 3 and 4 instead.

10. It loses the possible root 0.

Division assumes x is nonzero.

11. 7 cm.

The equation gives ± 7, but a physical side is positive.

12. 4 by 5 units.

$(x + 5)(x - 4) = 0$; reject -5.

13. No.

Roots may be irrational, as for $x^2 - 2 = 0$.

14. No.

The zero-product property requires the product to be zero.